BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 00, Number 0, Xxxx XXXX, Pages 000–000
S 0273-0979(XX)0000-0

BALLMANN, BRIDSON-HAEFLIGER, EBERLEIN

BRUCE KLEINER

The study of nonpositively curved spaces goes back to the discovery of hyperbolic space, the work of Hadamard around 1900, and Cartan's work in the 20's. These spaces play a significant role in many areas: Lie group theory, combinatorial and geometric group theory, dynamical systems, harmonic maps and vanishing theorems, geometric topology, Kleinian group theory, and Teichmüller theory. In some of these contexts –for instance in dynamics and in harmonic map theory – nonpositive curvature turns out to be the right condition to make things work smoothly, while in others such as Lie theory, 3-manifold topology, and Teichmüller theory, the basic objects of study happen to be nonpositively curved spaces. With so many closely related interdependent fields, nonpositive curvature has been a very active topic in the last twenty years. To get an idea of the scope of the activity, consider some of the highlights:

Harmonic maps: [Cor92, GS92, KS93, MSY93]
3-manifolds and Kleinian groups: [MS84, Gab92, CJ94, Ota96, Ota98, Min99, Min94, Can93, McM96, Kap01, Gab97, GMT]
Structure theory and rigidity: [BS87, BBE85, BBS85, EH90, BB95, Lee97].
High dimensional topology: [FH81, FJ93, CGM90].
Hyperbolic groups, quasi-conformal geometry/analysis: [Gro87, Pan89, Sel95, RS94, BM91, Bow98a, Bow98b, BP99, BP00, HK98].
Geometric/combinatorial group theory: [Gro87, DJ91, CD95, BM97, KL97a, KL97b].
Dynamics: [Cro90, Ota90, BCG95, BFK98]

One point of view which has been quite influential in recent years is that it is fruitful to work with "synthetic" conditions which are equivalent to nonpositive sectional curvature in the Riemannian case, rather than sectional curvature itself. Though this idea (and the analog for spaces with lower curvature bounds) goes back to A. D. Alexandrov, it was Gromov [Gro87] who brought it to the attention of a much wider

Received by the editors March 11, 2001.

audience in the 80's. The most popular alternate condition is a triangle comparison inequality which says that "sufficiently small geodesic triangles are at least as thin as corresponding Euclidean triangles". The precise version runs as follows. We say that a metric space X is a *Hadamard space* or $CAT(0)$ *space*[1] if

1. X is complete;
2. Every two points in X are joined by a geodesic segment (a path whose length equals the distance between its endpoints);
3. Whenever $x, y, z \in X$, $x', y', z' \in \mathbb{R}^2$ are isometric triples ($d_X(x,y) = d_{\mathbb{R}^2}(x',y')$, $d_X(y,z) = d_{\mathbb{R}^2}(y',z')$, $d_X(z,x) = d_{\mathbb{R}^2}(z',x')$), w lies on a geodesic segment joining y to z, and w' is the corresponding point on the Euclidean segment $\overline{y'z'}$ ($d_{\mathbb{R}^2}(w',y') = d_X(w,y)$), then $d_X(w,x) \leq d_{\mathbb{R}^2}(w',x')$.

We say that a metric space Z is an *Alexandrov space with nonpositive curvature*, or simply a *nonpositively curved space*, if for every $z \in Z$ there is an $r > 0$ so that the closed ball $\overline{B(z,r)}$ is a Hadamard space with respect to the induced metric. Work of Cartan [Car88] implies that a Riemannian manifold has nonpositive sectional curvature iff it defines an Alexandrov space with nonpositive curvature. A Riemannian manifold is a Hadamard space iff it is a Hadamard manifold (a complete, simply connected manifold with nonpositive sectional curvature).

Working with Alexandrov spaces rather than nonpositively curved manifolds has several advantages. Most of the foundational material carries over to the more general setting with only minor modifications, and the proofs, which are no longer allowed to use objects that depend on smooth structure (the Levi-Civita connection, Jacobi fields), sometimes become simpler. In this more general context one still has a tangent cone associated with each point of a Hadamard space X, and a "tangent cone at infinity" – the Tits cone $C_T X$ (named after Jacques Tits); these tangent cones are Hadamard spaces, and they play a leading role in the theory. The Tits cone is almost never (isometric to) a Riemannian manifold, even when X itself is a Riemannian manifold. Many important examples of Hadamard spaces, especially examples from geometric group theory, are non-Riemannian. For instance simplicial trees (and more generally $\mathbb{R}$-trees), and Euclidean and hyperbolic Tits buildings. A connected finite 2-complex built by gluing together Euclidean triangles along their edges defines a space with

[1]The letters C, A, and T stand for Cartan, Alexandrov, and Toponogov. The term *Hadamard space* was introduced in [Bal95], since Hadamard spaces are generalizations of Hadamard manifolds.

nonpositive curvature iff each vertex link, when endowed with the angle metric, is a graph with no cycles of length $< 2\pi$; this construction already gives an abundance of interesting examples and is important in the theory of small cancellation groups.

The books under review have several common themes. All three base their development on the geometry of geodesics and distance functions, and, for the most part, they work toward results which apply to a broad class of nonpositively curved spaces rather than focussing on special classes of spaces. The basic object of study is a Hadamard space X with an isometric group action $\Gamma \times X \to X$; one typically gets such an action by taking the deck group action for the universal covering of a nonpositively curved space. Under appropriate conditions one gets a strong relationship between algebraic structure in Γ (abelian subgroups, product structure, centralizers) and geometric structure in X (flat convex subspaces, product structure, convex subspaces with Euclidean factors). Results of this type go back to [GW71, LY72, Ebe82]. The proofs depend crucially on the fact that the curvature is allowed to be zero, i.e. flat subspaces are allowed; in fact many of the most striking results in the subject rely on rigid behavior associated with flat subspaces. Another key isssue in the books is the global behavior of geodesics; this comes into play in three closely related guises – the geodesic flow GX, the boundary at infinity and the Tits boundary. The geodesic flow GZ of a metric space Z is the set of unit speed locally geodesic paths in Z, topologized by uniform convergence on compact subsets, and equipped with the $\mathbb{R}$-action $\mathbb{R}\times GZ \to GZ$ defined by precomposition with translation in $\mathbb{R}$. (When Z is a complete Riemannian manifold the $\mathbb{R}$-action $\mathbb{R} \times GZ \to GZ$ is equivalent to the usual geodesic flow $\mathbb{R}\times SZ \to SZ$ on the unit tangent bundle SZ.) The geodesic flow of a complete nonpositively curved space Z relates directly with the fundamental group: each periodic orbit of the flow determines a nontrivial conjugacy class in the fundamental group, and two periodic orbits determine distinct conjugacy classes unless the corresponding maps $S^1 \to M$ bound a locally isometric map $S^1 \times [0, L] \to M$ of a flat cylinder into M. If X is a Hadamard space, then one can use the asymptotic behavior of geodesics to define a *boundary at infinity* $\partial_\infty X$ as follows. One says that two unit speed geodesic rays $\alpha_1 : [0,\infty) \to X$ and $\alpha_2 : [0,\infty) \to X$ are *asymptotic* if the distance $d_X(\alpha_1(t), \alpha_2(t))$ is bounded independent of t. The relation of being asymptotic is an equivalence relation on the set of unit speed rays. As a set, the boundary at infinity $\partial_\infty X$ is the collection of asymptote classes of unit speed geodesic rays; to define a topology on this set, one observes that $\partial_\infty X$

can be identified with the set of unit speed rays leaving any given basepoint $p \in X$, and the latter has a natural topology – the topology of uniform convergence on compact sets; finally one verifies that the topology this induces on $\partial_\infty X$ is independent of the choice of basepoint p. When X is an n-dimensional Hadamard manifold $\partial_\infty X$ is homeomorphic to S^{n-1}. The isometry group of X has an induced action on $\partial_\infty X$ by homeomorphisms. When $\Gamma \times X \to X$ is the deck group action for the universal covering of a compact nonpositively curved space Z, the induced action $\Gamma \times \partial_\infty X \to \partial_\infty X$ is one of the key tools for analyzing the geodesic flow of Z and the structure of Γ. It is also used in the proof of rigidity theorems like [Mos73, BCG95, Pan89, BP00]. To define the Tits boundary of a Hadamard space, one starts with the set $\partial_\infty X$, and defines the distance between the asymptote classes of two unit speed rays α_1 and α_2 to be the "asymptotic angle of divergence", i.e. $2\arcsin(\frac{\rho}{2})$ where $\rho := \lim_{t\to\infty} \frac{1}{t} d(\alpha_1(t), \alpha_2(t))$ (this limit always exists for unit speed rays in Hadamard spaces, and depends only on the asymptote classes of the rays). This distance function (which is called the Tits angle metric) usually defines a different topology on the set $\partial_\infty X$ than the one mentioned above – the topology usually does not have a countable basis; the metric space it defines is denoted $\partial_T X$. The Tits boundary $\partial_T X$ registers asymptotically Euclidean structure in X: Euclidean subspaces $F^k \subset X$ produce round spheres $S^{k-1} \subset \partial_T X$, and under mild assumptions a partial converse holds.

A common objective of the books by Ballmann and Eberlein is the rank rigidity theorem. This is a structure theorem for complete, finite volume, nonpositively curved Riemannian manifolds M, and is the centerpiece of the theory[2]. It says that if $\tilde{M} = X_0 \times X_1 \times \ldots X_k$ is the de Rham decomposition of the universal cover of M (X_0 is the Euclidean factor), then each X_i is either an irreducible symmetric space of noncompact type of rank at least two, or else it contains a geodesic which does not bound a flat half plane (a subset isometric to $\{(x, y) \in \mathbb{R}^2 \mid y \geq 0\}$) in X_i. The theorem was first proved in this precise form in [EH90], slightly extending the results in the earlier papers [Bal85, BBE85, BBS85, BS87]. The theorem, when combined with earlier work [Bal82], has many implications (for manifolds M as above) with no other known proofs: periodic orbits of the geodesic flow are dense (when the fundamental group is finitely generated); $\pi_1(M)$ contains nonabelian free groups unless M is a compact flat manifold; the number of distinct primitive free homotopy classes of maps $S^1 \to M$

[2]The actual statement is somewhat more general than this; I have only stated the finite volume case for simplicity.

having representatives with length at most L grows exponentially with L, unless M is flat; the geodesic flow of M has a dense orbit unless the universal cover $\tilde{M}$ splits as a Riemannian product or is an irreducible symmetric space of rank at least two. Preparations for the proof of the rank rigidity theorem occupy a good fraction of both volumes.

Ballmann's book is by far the shortest of the three, and runs under 100 pages (even including the 15 page appendix by Misha Brin). The reader is assumed to have some familiarity with basic concepts of Riemannian geometry – geodesics, Jacobi fields, sectional curvature, and comparison theorems. The first two chapters cover the basic facts about nonpositively curved spaces, like the Cartan-Hadamard theorem, Busemann functions, the boundary at infinity, the Tits boundary, and the classification of isometries. Chapter III ("Weak hyperbolicity") is based on the author's paper [Bal82]; it shows that if $\Gamma \times X \to X$ is an isometric action on a locally compact Hadamard space and there is a geodesic $\gamma \subset X$ which does not bound a flat half plane, then, provided $\Gamma \times X \to X$ satisfies the "duality condition" (this will hold if $\Gamma \times X \to X$ is the deck group action for the universal covering of a compact Riemannian manifold), then X behaves in many respects like a space with negative curvature. Chapter III also shows that the Dirichlet problem at infinity is solvable for actions $\Gamma \times X \to X$ as above. Chapter IV proves the rank rigidity theorem. There is an appendix by Misha Brin which proves the ergodicity of geodesic flows of compact Riemannian manifolds with strictly negative curvature. This book packs an amazing amount of material into 100 pages without compromising readability. Anyone who wants to learn a proof of rank rigidity with a minimal time commitment, or anyone looking for a concise discussion of Hadamard space geometry should find this book rewarding.

Eberlein covers much of the same ground as Ballmann (though he works with Riemannian manifolds rather than Alexandrov spaces), as well as several other topics. Overall his treatment is much more detailed, and his style is more expansive. The prerequisites are the same Riemannian geometry that Ballmann requires plus some Lie group theory; everything is reviewed in the first chapter. He has an extensive discussion of symmetric spaces of noncompact type which ties together notions from Lie theory and geometry. Aside from their importance in the rank rigidity theorem (not to mention their importance in Lie group theory), symmetric spaces offer intricate examples illustrating all the general theory of Hadamard spaces. To my knowledge there is no comparable treatment available in the literature – this will be very valuable to anyone working in the field. Eberlein also proves part of the Mostow rigidity theorem – the cases where one can avoid the quasi-conformal geometry in rank 1 – following Mostow's original proof; this was part of his motivation for the detailed discussion of symmetric space geometry. In the penultimate chapter he collects consequences of rank rigidity and Mostow rigidity. This book has already become a standard reference for nonpositively curved manifolds. It would be a good source for a second or third year graduate course on nonpositive curvature.

Compared to the other two authors, Bridson and Haefliger cover a broad swath of terrain, treating the foundations for many different topics instead of working toward a small number of difficult theorems. The book is written so as to be accessible to first year graduate students – no Riemannian geometry or Lie group theory is required. The treatment of Alexandrov spaces is much more extensive than Ballmann's, and runs over 300 pages. Here they have interspersed material on Gromov-Hausdorff convergence, ultralimits, quasi-isometries and quasi-isometry invariants. There is a section on the geometry of the symmetric space for $GL(n,\mathbb{R})$; this serves to introduce the reader to the ideas by way of examples and does not attempt a systematic treatment. The third part of the book has chapters on Gromov hyperbolicity, nonpositive curvature and group theory, and complexes of groups/orbifolds. The main objective of the last two chapters is the theorem (due to Haefliger) that, roughly speaking, an orbifold is developable (i.e. can be obtained as the quotient orbifold for a group action) provided it admits a non-positively curved structure. The proof is similar to the proof of the Cartan-Hadamard theorem. This book will be a useful reference for Alexandrov space geometry. The exposition has a gentle pace, making it very suitable for a reading course in geometric group theory.

The subject has advanced tremendously in recent years, yet many fundamental questions remain. I would like to close with three well-known open problems:

- *The flat closing problem.* If Z is a compact nonpositively curved space, and the universal cover $\tilde{Z}$ contains a subset isometric to $\mathbb{R}^2$, does the fundamental group $\pi_1(Z)$ contain a copy of $\mathbb{Z}^2$? This is known only when Z is a 3-dimensional Riemannian manifold, or a Riemannian manifold with a real analytic metric. It is open for finite 2-complexes built from Euclidean squares.
- *Rank rigidity for singular spaces.* Is there a version of rank rigidity for compact nonpositively curved spaces? For example, if Z is compact, nonpositively curved, and has extendible geodesics (every locally geodesic segment can be extended to a locally geodesic

path $\mathbb{R} \to Z$), is it true that one of the following holds: (1) $\partial_T \tilde{Z}$ is disconnected; (2) $\tilde{Z}$ is a product; (3) $\tilde{Z}$ is a symmetric space; (4) $\tilde{Z}$ is a Euclidean building? This is known for piecewise Euclidean 2-complexes [BB95].

- *Tits alternative.* If Z is compact and nonpositively curved, is it true that every subgroup of $\pi_1(Z)$ either contains a free nonabelian subgroup or finite index abelian subgroup?

References

[Bal82] Werner Ballmann, *Axial isometries of manifolds of nonpositive curvature*, Math. Ann. **259** (1982), no. 1, 131–144.

[Bal85] Werner Ballmann, *Nonpositively curved manifolds of higher rank*, Ann. of Math. (2) **122** (1985), no. 3, 597–609.

[Bal95] W. Ballmann, *Lectures on spaces of nonpositive curvature*, Birkhäuser Verlag, Basel, 1995.

[BB95] Werner Ballmann and Michael Brin, *Orbihedra of nonpositive curvature*, Inst. Hautes Études Sci. Publ. Math. (1995), no. 82, 169–209 (1996).

[BBE85] Werner Ballmann, Misha Brin, and Patrick Eberlein, *Structure of manifolds of nonpositive curvature. I*, Ann. of Math. (2) **122** (1985), no. 1, 171–203.

[BBS85] Werner Ballmann, Misha Brin, and Ralf Spatzier, *Structure of manifolds of nonpositive curvature. II*, Ann. of Math. (2) **122** (1985), no. 2, 205–235.

[BCG95] G. Besson, G. Courtois, and S. Gallot, *Entropies et rigidités des espaces localement symétriques de courbure strictement négative*, Geom. Funct. Anal. **5** (1995), no. 5, 731–799.

[BFK98] D. Burago, S. Ferleger, and A. Kononenko, *Uniform estimates on the number of collisions in semi-dispersing billiards*, Ann. of Math. (2) **147** (1998), no. 3, 695–708.

[BM91] Mladen Bestvina and Geoffrey Mess, *The boundary of negatively curved groups*, J. Amer. Math. Soc. **4** (1991), no. 3, 469–481.

[BM97] Marc Burger and Shahar Mozes, *Finitely presented simple groups and products of trees*, C. R. Acad. Sci. Paris Sér. I Math. **324** (1997), no. 7, 747–752.

[Bow98a] Brian H. Bowditch, *Cut points and canonical splittings of hyperbolic groups*, Acta Math. **180** (1998), no. 2, 145–186.

[Bow98b] Brian H. Bowditch, *A topological characterisation of hyperbolic groups*, J. Amer. Math. Soc. **11** (1998), no. 3, 643–667.

[BP99] Marc Bourdon and Hervé Pajot, *Poincaré inequalities and quasiconformal structure on the boundary of some hyperbolic buildings*, Proc. Amer. Math. Soc. **127** (1999), no. 8, 2315–2324.

[BP00] Marc Bourdon and Hervé Pajot, *Rigidity of quasi-isometries for some hyperbolic buildings*, Comment. Math. Helv. **75** (2000), no. 4, 701–736.

[BS87] Keith Burns and Ralf Spatzier, *Manifolds of nonpositive curvature and their buildings*, Inst. Hautes Études Sci. Publ. Math. (1987), no. 65, 35–59.

[Can93] Richard D. Canary, *Ends of hyperbolic* 3*-manifolds*, J. Amer. Math. Soc. **6** (1993), no. 1, 1–35.

[Car88] Élie Cartan, *Leçons sur la géométrie des espaces de Riemann*, Éditions Jacques Gabay, Sceaux, 1988, Reprint of the second (1946) edition.

[CD95] Ruth M. Charney and Michael W. Davis, *Strict hyperbolization*, Topology **34** (1995), no. 2, 329–350.

[CGM90] Alain Connes, Mikhaïl Gromov, and Henri Moscovici, *Conjecture de Novikov et fibrés presque plats*, C. R. Acad. Sci. Paris Sér. I Math. **310** (1990), no. 5, 273–277.

[CJ94] Andrew Casson and Douglas Jungreis, *Convergence groups and Seifert fibered* 3*-manifolds*, Invent. Math. **118** (1994), no. 3, 441–456.

[Cor92] Kevin Corlette, *Archimedean superrigidity and hyperbolic geometry*, Ann. of Math. (2) **135** (1992), no. 1, 165–182.

[Cro90] Christopher B. Croke, *Rigidity for surfaces of nonpositive curvature*, Comment. Math. Helv. **65** (1990), no. 1, 150–169.

[DJ91] Michael W. Davis and Tadeusz Januszkiewicz, *Hyperbolization of polyhedra*, J. Differential Geom. **34** (1991), no. 2, 347–388.

[Ebe82] Patrick Eberlein, *A canonical form for compact nonpositively curved manifolds whose fundamental groups have nontrivial center*, Math. Ann. **260** (1982), no. 1, 23–29.

[EH90] Patrick Eberlein and Jens Heber, *A differential geometric characterization of symmetric spaces of higher rank*, Inst. Hautes Études Sci. Publ. Math. (1990), no. 71, 33–44.

[FH81] F. T. Farrell and W. C. Hsiang, *On Novikov's conjecture for nonpositively curved manifolds. I*, Ann. of Math. (2) **113** (1981), no. 1, 199–209.

[FJ93] F. T. Farrell and L. E. Jones, *Topological rigidity for compact nonpositively curved manifolds*, Differential geometry: Riemannian geometry (Los Angeles, CA, 1990), Amer. Math. Soc., Providence, RI, 1993, pp. 229–274.

[Gab92] David Gabai, *Convergence groups are Fuchsian groups*, Ann. of Math. (2) **136** (1992), no. 3, 447–510.

[Gab97] David Gabai, *On the geometric and topological rigidity of hyperbolic* 3*-manifolds*, J. Amer. Math. Soc. **10** (1997), no. 1, 37–74.

[GMT] D. Gabai, R. Meyerhoff, and N. Thurston, *Homotopy hyperbolic 3-manifolds are hyperbolic*, Annals of Math, to appear.

[Gro87] M. Gromov, *Hyperbolic groups*, Essays in group theory, Springer, New York, 1987, pp. 75–263.

[GS92] Mikhail Gromov and Richard Schoen, *Harmonic maps into singular spaces and p-adic superrigidity for lattices in groups of rank one*, Inst. Hautes Études Sci. Publ. Math. (1992), no. 76, 165–246.

[GW71] Detlef Gromoll and Joseph A. Wolf, *Some relations between the metric structure and the algebraic structure of the fundamental group in manifolds of nonpositive curvature*, Bull. Amer. Math. Soc. **77** (1971), 545–552.

[HK98] Juha Heinonen and Pekka Koskela, *Quasiconformal maps in metric spaces with controlled geometry*, Acta Math. **181** (1998), no. 1, 1–61.

[Kap01] Michael Kapovich, *Hyperbolic manifolds and discrete groups*, Birkhäuser Boston Inc., Boston, MA, 2001.

[KL97a] Michael Kapovich and Bernhard Leeb, *Quasi-isometries preserve the geometric decomposition of Haken manifolds*, Invent. Math. **128** (1997), no. 2, 393–416.

[KL97b] Bruce Kleiner and Bernhard Leeb, *Rigidity of quasi-isometries for symmetric spaces and Euclidean buildings*, Inst. Hautes Études Sci. Publ. Math. (1997), no. 86, 115–197 (1998).

[KS93] Nicholas J. Korevaar and Richard M. Schoen, *Sobolev spaces and harmonic maps for metric space targets*, Comm. Anal. Geom. **1** (1993), no. 3-4, 561–659.

[Lee97] B. Leeb, *A characterization of irreducible symmetric spaces and euclidean buildings of higher rank by their asymptotic geometry*, Preprint, June, 1997.

[LY72] H. Blaine Lawson, Jr. and Shing Tung Yau, *Compact manifolds of nonpositive curvature*, J. Differential Geometry **7** (1972), 211–228.

[McM96] Curtis T. McMullen, *Renormalization and 3-manifolds which fiber over the circle*, Princeton University Press, Princeton, NJ, 1996.

[Min94] Yair N. Minsky, *On rigidity, limit sets, and end invariants of hyperbolic 3-manifolds*, J. Amer. Math. Soc. **7** (1994), no. 3, 539–588.

[Min99] Yair N. Minsky, *The classification of punctured-torus groups*, Ann. of Math. (2) **149** (1999), no. 2, 559–626.

[Mos73] G. D. Mostow, *Strong rigidity of locally symmetric spaces*, Princeton University Press, Princeton, N.J., 1973, Annals of Mathematics Studies, No. 78.

[MS84] John W. Morgan and Peter B. Shalen, *Valuations, trees, and degenerations of hyperbolic structures. I*, Ann. of Math. (2) **120** (1984), no. 3, 401–476.

[MSY93] Ngaiming Mok, Yum Tong Siu, and Sai-Kee Yeung, *Geometric superrigidity*, Invent. Math. **113** (1993), no. 1, 57–83.

[Ota90] Jean-Pierre Otal, *Le spectre marqué des longueurs des surfaces à courbure négative*, Ann. of Math. (2) **131** (1990), no. 1, 151–162.

[Ota96] Jean-Pierre Otal, *Le théorème d'hyperbolisation pour les variétés fibrées de dimension 3*, Astérisque (1996), no. 235, x+159.

[Ota98] Jean-Pierre Otal, *Thurston's hyperbolization of Haken manifolds*, Surveys in differential geometry, Vol. III (Cambridge, MA, 1996), Int. Press, Boston, MA, 1998, pp. 77–194.

[Pan89] Pierre Pansu, *Métriques de Carnot-Carathéodory et quasiisométries des espaces symétriques de rang un*, Ann. of Math. (2) **129** (1989), no. 1, 1–60.

[RS94] E. Rips and Z. Sela, *Structure and rigidity in hyperbolic groups. I*, Geom. Funct. Anal. **4** (1994), no. 3, 337–371.

[Sel95] Z. Sela, *The isomorphism problem for hyperbolic groups. I*, Ann. of Math. (2) **141** (1995), no. 2, 217–283.

Department of Mathematics, University of Michigan, Ann Arbor
E-mail address: bkleiner@math.lsa.umich.edu