Current Sheets, Reconnection and Adaptive Mesh Refinement

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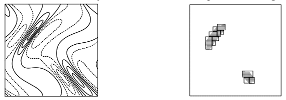
Adaptive Mesh Refinement Basics

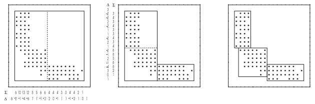
Two main parts
- Recursive integration of the grid hierarchy.
- Recursive regridding to build the grid hierarchy and to dynamically adjust it to the flow.

First Refinement
Example: 2D ideal MHD

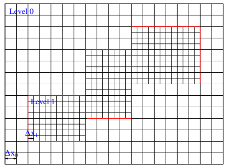
Current density — Marked points & new grids



Grid generation — compare Berger, Rigoutsos (1991)



Grid structure



2D ideal MHD

joint work with Diego Cordoba, IAS, Princeton

Equations

Vorticity $\quad \boldsymbol{\omega} = \nabla \times \mathbf{u}$,
Current density $\quad \mathbf{j} = \nabla \times \mathbf{B}$.

Two-dimensional:

$$\mathbf{u} = \mathbf{e}_z \times \nabla\varphi \ , \ \omega = (\nabla \times \mathbf{u})_z = \Delta\varphi \, ,$$
$$\mathbf{B} = \mathbf{e}_z \times \nabla\psi \ , \ j = (\nabla \times \mathbf{B})_z = \Delta\psi \, .$$

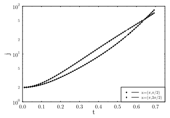
$\Longrightarrow$ Scalar equations

$$\partial_t\omega + \mathbf{u}\cdot\nabla\omega = \mathbf{B}\cdot\nabla j \, ,$$
$$\partial_t\psi + \mathbf{u}\cdot\nabla\psi = 0 \, .$$

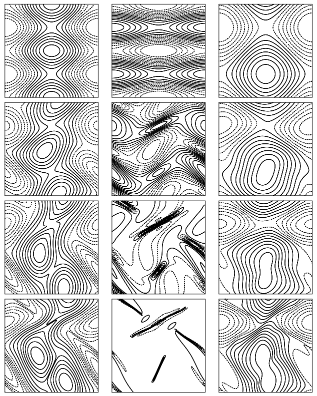
Numerical studies

Initial conditions: $\quad \psi_0(x_1,x_2) = 2\cos(x_1) - \cos(x_2)$
$\quad \varphi_0(x_1,x_2) = 2(\cos(x_1) - \sin(x_2))$

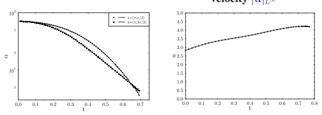
Absolute value of current density



Magnetic stream function (left), current density (middle) and kinetic stream function (right) at times $t = 0., \ 0.25, \ 0.51, \ 0.75$

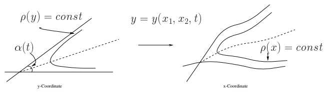


Angle — **Maximum norm of velocity $|\mathbf{u}|_{L^\infty}$**



Analytical estimates

Hyperbolic saddle



Theorem 1 *Let the stream function $\psi(x_1,x_2,t)$ be a smooth solution of (1) and (1) and constant along the hyperbolas defined above, $\psi = \psi(\rho,t)$, in a neighborhood U for $t \in [0,T_*)$. Assume for each fixed t, that ψ is not constant on any disc in U and $u \in L^\infty$ in U for $t \in [0,T_*]$. Then $\lim_{t\to T_*}$ exists and is not 0.*

Theorem 2 *Let $\psi(x_1,x_2,t)$, β, δ, U and y_i be as in Theorem 1, but with $T_* = \infty$. Assume the C^2 semi-norms of y_i are bounded for all time $t \in [0,\infty)$. Then*

$$|\log\frac{1}{\alpha(t)}| \le C_1\|\mathbf{u}\|_{L^\infty}t + C_2.$$

2D resistive MHD

joint work with Hank Strauss, CIMS

Equations

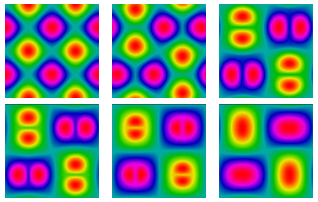
$$\partial_t\omega + \mathbf{u}\cdot\nabla\omega = \mathbf{B}\cdot\nabla j + \nu\Delta\omega \, ,$$
$$\partial_t\psi + \mathbf{u}\cdot\nabla\psi = \eta\Delta\psi \, .$$

Numerical results

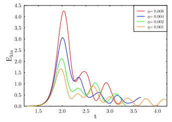
Initial conditions:

Ideal equilibrium: $\quad \psi_0(x_1,x_2) = \frac{1}{2}[\cos(2\pi x_1) - \cos(2\pi x_2)]$
and small perturbation: $\quad \varphi_0(x_1,x_2)$

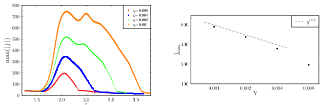
Magnetic stream function ψ
at times $t = 1.25, \ 1.85, \ 2.20, \ 2.35, \ 2.70, \ 3.32$



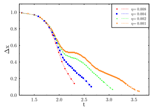
Kinetic energy



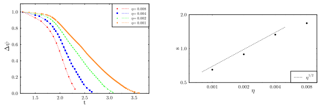
Absolute maximum of current density



Distance of O-points



Difference of magnetic flux between O- and X-points



2D MHD with electron inertia in the Ohm's law

Equations

$$\nu = \eta = 0, \ d_e = 0.3$$

$$\partial_t\omega + \mathbf{u}\cdot\nabla\omega = \mathbf{B}\cdot\nabla j \, ,$$
$$\partial_t(\psi - d_e^2 j) + \mathbf{u}\cdot\nabla(\psi - d_e^2 j) = 0 \, .$$

Numerical results

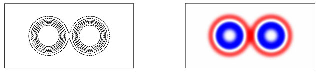
Initial conditions:

$$\omega(\mathbf{x}) = 0 \, ,$$
$$\psi(\mathbf{x}) = \exp\left[\{(\mathbf{x}-\mathbf{x_1})^2/4\}\right] + \exp\left[\{(\mathbf{x}-\mathbf{x_2})^2/4\}\right]$$

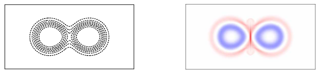
with $\mathbf{x}_1 = (1.4(\frac{\pi}{2}+0.2), 0)$ and $\mathbf{x}_2 = (1.4(\frac{\pi}{2}+0.2), 0)$.

Magnetic flux function — Current density

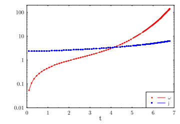
at time $t = 0$



at time $t = 6.4$

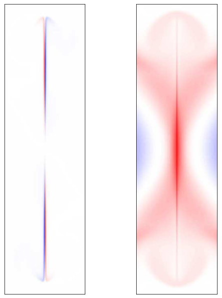


Current density and vorticity



Sheet structure enlarged by a factor 12

Vorticity — Current density



3D ideal MHD

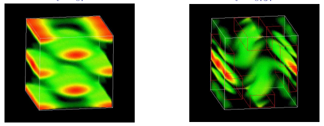
Equations

$$\partial_t\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = \mathbf{B}\cdot\nabla\mathbf{B} \, ,$$
$$\partial_t\mathbf{B} + \mathbf{u}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{u} \, ,$$
$$\nabla\cdot\mathbf{u} = 0 \, ,$$
$$\nabla\cdot\mathbf{B} = 0 \, .$$

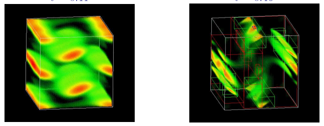
Numerical results

Volume renderings of current density

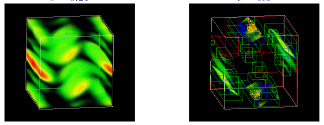
$t = 0.$ — $t = 0.37$



$t = 0.11$ — $t = 0.49$



$t = 0.24$ — $t = 0.6$



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Joint work with D. Cordoba, IAS Princeton, R. Grauer, Universität Düsseldorf, and H. Strauss, CIMS.