### Complex Variables II

## SYLLABUS

#### **Textbook:**

L. Ahlfors: Complex Analysis

### **Recommended:**

- J. Brown and R. Churchill: Complex Variables
- Z. Nehari: Conformal Mapping
- M. Spiegel: Schaum's Outline: Complex Variables

### I. Properties of Analytic Functions

- (i) The argument principle and Rouche's theorem
- (ii) The local mapping, the maximum principle, Schwarz's lemma
- (iii) Harmonic functions: maximum principle, Poisson's formula, Schwarz's theorem

#### **II. Families of Analytic Functions**

- (i) Uniform Convergence, Weierstrass' theorem
- (ii) Normal families, equicontinuity, Arzela's (Montel's) theorem

## **III. Infinite Sums and Products**

- (i) Analyticity at  $\infty$ , polynomials, rational functions
- (ii) Meromorphic functions, the Mittag-Leffler theorem
- (iii) Infinite products of numbers and functions, uniform convergence
- (iv) Canonical products, the genus and order of entire functions
- (v) The Gamma Function: infinite product and integral representations

# IV. Conformal Mapping

- (i) Properties of simple mappings: powers, the exponential function, the bilinear map
- (ii) Univalent maps, the Riemann Mapping Theorem
- (iii) Continuity of maps at the domain boundary, the Schwartz Reflection Principle
- (iv) Mapping of Polygons (Schwartz-Christoffel), elliptic integrals

# V. Analytic Continuation

- (i) Function elements, standard procedure of analytic continuation, continuation along arcs, points of continuation and the natural boundary of analytic functions
- (ii) The monodromy theorem
- (iii) Picard's theorem