1

$\vec{u} = \langle 1, 1, \sqrt{2} \rangle$

$\vec{v} = \langle 1, 1, 0 \rangle$

(a) $\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & \sqrt{2} \\ 1 & 1 & 0 \end{vmatrix} = -\sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j} = \langle -\sqrt{2}, \sqrt{2}, 0 \rangle$

$= \sqrt{2}\langle -1, 1, 0 \rangle$

(b) $\vec{u} \cdot \vec{v} = 1 + 1 + 0 = 2$

(c) $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

$|\vec{u}| = \sqrt{1+1+2} = 2$

$|\vec{v}| = \sqrt{1+1+0} = \sqrt{2}$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{2}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}$

(d) $\text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|}\right) \dfrac{\vec{v}}{|\vec{v}|} = \left(\dfrac{2}{\sqrt{2}}\right) \dfrac{\langle 1, 1, 0 \rangle}{\sqrt{2}} = \langle 1, 1, 0 \rangle$

(e) Volume $= |\langle 1, 0, 0 \rangle \cdot \vec{u} \times \vec{v}|$

$= |\langle 1, 0, 0 \rangle \cdot \langle -\sqrt{2}, \sqrt{2}, 0 \rangle|$

$= \sqrt{2}$

(2)

(a) $\vec{k} \times \vec{i} = \vec{j}$

(b) $(-\vec{j}) \times \vec{k} = -\vec{i}$

(c) $(-\vec{j}) \cdot \vec{k} = 0$

(d) $\vec{k} \times (-\vec{j}) = \vec{i}$

(e) $\vec{i} \times (\vec{k} + \vec{j}) = \vec{i} \times \vec{k} + \vec{i} \times \vec{j} = -\vec{j} + \vec{k} = \langle 0, -1, 1 \rangle$

Could solve these using right hand rule or using definition of cross product. (Dot product for (c).)

③

$$\vec{r}(t) = \left\langle \tfrac{1}{6}t^3, t, \tfrac{1}{2}t^2 \right\rangle$$

$$\vec{r}\,'(t) = \left\langle \tfrac{1}{2}t^2, 1, t \right\rangle$$

$$|\vec{r}\,'(t)| = \sqrt{\tfrac{1}{4}t^4 + 1 + t^2} = \sqrt{\left(\tfrac{1}{2}t^2+1\right)^2} = \tfrac{1}{2}t^2+1$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\left\langle \tfrac{1}{2}t^2, 1, t \right\rangle}{\tfrac{1}{2}t^2+1}$$

(a) $$\vec{T}\,'(t) = \frac{\langle t, 0, 1 \rangle}{\tfrac{1}{2}t^2+1} - \frac{\left\langle \tfrac{1}{2}t^2, 1, t \right\rangle t}{\left(\tfrac{1}{2}t^2+1\right)^2}$$

$$= \frac{\left\langle \tfrac{1}{2}t^3+t, 0, \tfrac{1}{2}t^2+1 \right\rangle - \left\langle \tfrac{1}{2}t^3, t, t^2 \right\rangle}{\left(\tfrac{1}{2}t^2+1\right)^2}$$

$$= \frac{\left\langle t, -t, -\tfrac{1}{2}t^2+1 \right\rangle}{\left(\tfrac{1}{2}t^2+1\right)^2}$$

$$|\vec{T}\,'(t)| = \frac{\sqrt{t^2+t^2+\left(-\tfrac{1}{2}t^2+1\right)^2}}{\left(\tfrac{1}{2}t^2+1\right)^2} = \frac{\sqrt{2t^2+\tfrac{1}{4}t^4-t^2+1}}{\left(\tfrac{1}{2}t^2+1\right)^2}$$

$$= \frac{\sqrt{\tfrac{1}{4}t^4+t^2+1}}{\left(\tfrac{1}{2}t^2+1\right)^2} = \frac{\tfrac{1}{2}t^2+1}{\left(\tfrac{1}{2}t^2+1\right)^2} = \frac{1}{\tfrac{1}{2}t^2+1}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\langle t, -t, -\frac{1}{2}t^2+1 \rangle}{\frac{1}{2}t^2+1}$$

$$\text{(b)} \quad \kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{(\frac{1}{2}t^2+1)^{-1}}{\frac{1}{2}t^2+1} = \frac{1}{(\frac{1}{2}t^2+1)^2}$$

$$\text{(c)} \quad \vec{T}(0) = \langle 0, 1, 0 \rangle = \vec{j}$$

$$\vec{N}(0) = \langle 0, 0, 1 \rangle = \vec{k}$$

$$\vec{B}(0) = \vec{T}(0) \times \vec{N}(0) = \vec{j} \times \vec{k} = \vec{i} = \langle 1, 0, 0 \rangle$$

(4) (a) False – could be a saddle point

(b) True – $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$ and $\vec{b}$

(c) Thrown out – would be true if $\frac{dz}{dt}$ were replaced by $\frac{\partial z}{\partial t}$

(d) True – this is one reason why it is useful to parameterize a curve with arc length
– can show it using the definition of arclength

(e) True – this was shown in class (and in the book)

⑤

(a) $f(x,y) = -xye^{-x^2-y^2}$

$f(0,y) = 0$

$f(x,0) = 0$

$\boxed{4}$

(b) $f(x,y) = \dfrac{-3x}{x^2+y^2+1}$

$f(0,y) = 0$

$\boxed{8}$

(c) $f(x,y) = y^2 - x^2$

$f(x,x) = 0$

$f(x,-x) = 0$

$\boxed{1}$

(d) $f(x,y) = \dfrac{x-y}{x^2+y^2+1}$

$f(x,x) = 0$

$f(x,-x) = \dfrac{2x}{2x^2+1}$ ← not oscillating like 7

$\boxed{3}$

(e) $f(x,y) = \sin(x-y)$

$f(0,y) = \sin(-y)$

$f(x,0) = \sin x$

$f(x,x) = 0$

$\boxed{7}$

(6)

(a) $x^2+2y^2+z^2=4$ $\qquad (x_0,y_0,z_0)=(1,-1,1)$

$$F(x,y,z)=x^2+2y^2+z^2=4$$

$$F_x = 2x$$
$$F_y = 4y$$
$$F_z = 2z$$

Tangent plane equation takes the form:

$$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+F_z(x_0,y_0,z_0)(z-z_0)=0$$

$$2(x-1)-4(y+1)+2(z-1)=0$$

or $2x-4y+2z-8=0$

or $z=-x+2y+4$

7

(a) $f(x,y) = x^4 + y^4 - 4xy + 1$

$f_x = 4x^3 - 4y = 0$

$f_y = 4y^3 - 4x = 0$

Critical points: $(0,0), (1,1), (-1,-1)$

(b) $f_{xx} = 12x^2$

$f_{yy} = 12y^2$

$f_{xy} = -4$

(i) $f_{xx}(0,0) = 0$

$D(0,0) = f_{xx}(0,0) f_{yy}(0,0) - [f_{xy}(0,0)]^2$

$= 0 - 16 < 0$

$(0,0)$ is a saddle point

(ii) $f_{xx}(1,1) = 12$

$D(1,1) = 12 \cdot 12 - 16 = 144 - 16 = 128 > 0$

$(1,1)$ is a local min

(iii) $f_{xx}(-1,-1) = 12$

$D(-1,-1) = 128 > 0$

$(-1,-1)$ is a local min