Calculus II: Final Exam
December 19, 2008

Omit one question. When you are done, write "omitted number ____" on the front of the exam book. Otherwise, #11 will not be graded. The questions are to be answered in order. Leave space if you want to delay answering a question. Calculators and cell phones are not to be in sight.

1. \[ \int e^x \cos x \ dx = \]

2. \[ \int \frac{x^2}{(4 - x^2)^{\frac{3}{2}}} \ dx = \]

3. \[ \int \frac{2x}{x^2 + 2x + 5} \ dx = \]

4. Find the volume of the solid obtained by rotating about the y-axis the region bounded by \( y^2 = x \) and \( x = 2y \).

5. Solve for \( y \):
\[
\frac{dy}{dx} = \frac{\cos x}{y^2}, \quad y(0) = 1.
\]

6. Convergent or divergent? Indicate the test that you have used.

   (a) \[ \sum_{n=0}^{\infty} \frac{n^2 + 1}{n^4 + 2} \]

   (b) \[ \sum \frac{2^n}{n!} \]

7. Find a power series expansion at \( x = 0 \) for \( \frac{x}{4 + x^2} \). What is the radius of convergence? What is the interval of convergence?
8. Evaluate the indefinite integral \( \int \frac{\sin x}{x} \, dx \) as an infinite series.

9. Find the first 3 non-zero terms of the Taylor series of \( \frac{x}{\cos x} \) at \( x = 0 \).

10. For the polar curve \( r = 1 + 2 \cos \theta \):

   (a) Find the cartesian coordinates of the point where \( \theta = \frac{3\pi}{4} \).

   (b) Find the slope of the tangent line at \( \theta = \frac{\pi}{4} \). (Note: \( \cos \theta = \sin \theta = \frac{\sqrt{2}}{2} \) for \( \theta = \frac{\pi}{4} \).)

11. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) for the parametric curve \( x = 1 + t^2, \ y = t - t^3 \).

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**List of Formulae**

\[
\begin{align*}
\frac{d}{dx} \tan x & = \sec^2 x \\
\frac{d}{dx} \sec x & = \sec x \cdot \tan x \\
\sin^2 x & = \frac{1 - \cos(2x)}{2} \\
\cos x & = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \\
e^x & = \sum_{n=0}^{\infty} \frac{x^n}{n!}
\end{align*}
\]

\[
\begin{align*}
\frac{d}{dx} \cot x & = -\csc^2 x \\
\sec^2 x & = 1 + \tan^2 x \\
\cos^2 x & = \frac{1 + \cos(2x)}{2} \\
\sin x & = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}
\end{align*}
\]