Professor Bremser

Calculus II: Final Exam

December 19, 2008

Omit one question. When you are done, write "omitted number ____" on the front of the exam book. Otherwise, #11 will not be graded. The questions are to be answered in order. Leave space if you want to delay answering a question. Calculators and cell phones are not to be in sight.

1. $$\int e^x \cos x \, dx =$$

2. $$\int \frac{x^2}{(4-x^2)^{\frac{3}{2}}} \, dx =$$

3. $$\int \frac{2x}{x^2+2x+5} \, dx =$$

4. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y^2 = x$ and $x = 2y$.

5. Solve for y:
$$\frac{dy}{dx} = \frac{\cos x}{y^2}, \qquad y(0) = 1.$$

6. Convergent or divergent? Indicate the test that you have used.

   (a)
   $$\sum_{n=0}^{\infty} \frac{n^2+1}{n^4+2}$$

   (b)
   $$\sum \frac{2^n}{n!}$$

7. Find a power series expansion at $x = 0$ for $\frac{x}{4+x^2}$. What is the radius of convergence? What is the interval of convergence?

8. Evaluate the indefinite integral $\int \frac{\sin x}{x}\, dx$ as an infinite series.

9. Find the first 3 non-zero terms of the Taylor series of $\frac{x}{\cos x}$ at $x = 0$.

10. For the polar curve $r = 1 + 2\cos\theta$:

    (a) Find the cartesian coordinates of the point where $\theta = \frac{3\pi}{4}$.

    (b) Find the slope of the tangent line at $\theta = \frac{\pi}{4}$. (Note: $\cos\theta = \sin\theta = \frac{\sqrt{2}}{2}$ for $\theta = \frac{\pi}{4}$.)

11. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the parametric curve $x = 1 + t^2$, $y = t - t^3$.

List of Formulae

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \cdot \tan x \qquad \sec^2 x = 1 + \tan^2 x$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$