Math for Economics III
New York University
Final Exam, Fall 2014

Name:______________________________ Recitation Section:____________________

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- The exam is closed book. You are not allowed to use a calculator nor consult any notes while taking the exam.
- This test has 7 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!
- The exam time limit is one hour and 50 minutes.

Good luck!!!

SCORES

Question 1(15 pts)	
Question 2(15 pts)	
Question 3(12 pts)	
Question 4(12 pts)	
Question 5(20 pts)	
Question 6(13 pts)	
Question 7(13 pts)	
TOTAL	

Question 1) (15 pts) Let $\vec{F} = x\vec{i} + e^{2y}\vec{j}$ and let C be the arc of the curve $\mathbf{x} = \mathbf{e^y}$ from (1,0) to (e,1).

(a) Parameterize the curve C.

(b) Compute the line integral $\int_C \vec{F} \cdot d\vec{r}$ directly by using the parametrization of the curve C you found in part (a).

(c) Show that $\vec{F}$ is a conservative vector field and find a potential function, f, namely a function such that $\text{grad} f = \vec{F}$.

(d) Use the Fundamental Theorem of Line Integrals to compute the same integral.

Question 2) (15 pts) In this problem you will either be asked to compute or setup an integral. If you use a theorem state why you can use it.

(a) Compute $\int_C xe^y\,dx + x^2e^y\,dy$, where C is consists of the line segment from (0,0) to (1,0) followed by the line segment from (1,0) to (1,1) followed by the arc of the parabola $y = x^2$ from (1,1) to (0,0).

(b) Setup an integral that is equivalent to

$$\iint_S (xy\vec{i} + (y^2 + e^{xz^2})\vec{j} + (\sin(xy) + (1-3y)z)\vec{k}) \cdot d\vec{S},$$

where S is the surface bounded by the planes $x = 0$, $y = 0$, $z = 0$, and by the tetrahedron $x + y + z = 1$. (You don't need to evaluate the integral but make sure you write the bounds for the integral.)

Question 3) (12 pts)

Determine whether each series converges or diverges. Please state the test you use and why you can use it.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{e^n - 1}{e^n + 1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdot \cdots \cdot (3n+2)}$

Question 4) (12 pts)

(a) Write the Taylor series about 0 for the function e^{-x^2} from a known Taylor series.

(b) Using part (a) write the Taylor series about 0 for the function $\dfrac{x}{e^{x^2}}$. Your final answer should be in sigma notation.

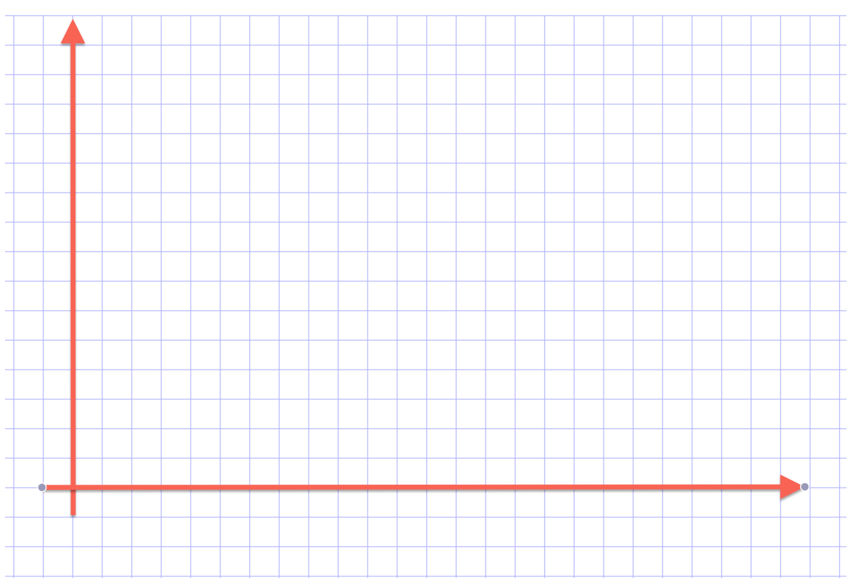
(c) Find the radius of convergence and interval of the convergence of the series from part (b).

Question 5) (20 pts)

A furniture manufacturing company is producing wooden tables and wooden chairs. To make one table, the company uses 30 board feet of wood and needs 5 hours. To make one chair, the company needs 20 board feet of wood and needs 10 hours. The company has in total 300 board feet of wood and has labor that can work at most for 110 hours. The company's unit profit for its tables is \$6 and for its chairs \$8. How many tables and chairs should the company produce in order to maximize its profit?

(a) State the linear programming problem, including the objective function, the inequality constraints and the non-negativity constraints.

(b) Solve this linear programming problem geometrically.



(c) Solve the problem using the Simplex Method.

(d) State the dual problem.

(e) Using your work in part (c) state the solutions to the dual problem. Write the interpretations of the numbers that optimizes the dual problem in the context of the furniture manufacturing company.

Question 6) (13 pts) We are given the following linear programming problem:

$$\min\ 2u_1 + u_2 \text{ subject to}$$

$$\begin{aligned} 3u_1 + u_2 &\geq 3 \\ u_1 + 2u_2 &\geq 4 \\ u_1 + 6u_2 &\geq 6 \\ u_1, u_2 &\geq 0 \end{aligned}$$

The optimal solution to this problem is $u_1 = \frac{2}{5}$ **and** $u_2 = \frac{9}{5}$**.**

(a) State the dual problem.

(b) Using the optimal solution you are given and complementary slackness find the optimal solution for the dual problem. Check the correctness of your answer by matching the optimal values of the primary and the dual problem.

Question 7) (13 pts)

Selin's turkish wrap truck dream is coming true. She has obtained a license to have three trucks in three neighborhoods of Manhattan: East Village, West Village, and Tribeca. She can finance only one truck in the city for now. Her best friend and her main competitor, Aisha, also has license to start a mexican burrito truck business in the same neighborhoods, and is similarly planning to open only one. Here is the payoff table for Selin! (The numbers represent the number of customers Selin will steal away from Aisha's truck in hundreds per day.)

		Aisha		
	Location	East Village	West Village	Tribeca
	East Village	4	1	3
Selin	West Village	3	2	5
	Tribeca	0	1	6

(a) Is there a saddle point in this game? If so what is it? Explain how you derived your answer and interpret your answer in the context of Aisha and Selin's business. What is the value of the game?

(b) Suppose Selin asked her economist friend to do research and to create a more accurate payoff table. She is given the table below.

		Aisha		
	Location	East Village	West Village	Tribeca
	East Village	0	-2	2
Selin	West Village	5	4	-3
	Tribeca	2	3	-4

i. You should note that this new payoff table has no saddle point. Using dominated strategies shrink the size of the payoff matrix as much as you can. (Hint: You can shrink it to a two by two matrix.)

ii. Since there is no saddle point this is a mixed strategy game. Write the linear programming corresponding to the payoff matrix from part (i) that would enable us to find the probabilities that Selin has to give to her strategies. Create the Simplex table for this problem. **You do not need to solve the problem.**