Lecture 13: Hard-to-Borrow Securities

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G63.2936.001

Spring Semester 2009
Hard-to-Borrow Stocks: 
Price dynamics and Option Valuation

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Mike Lipkin, Columbia University and Katama Trading

RiO, Hotel do Frade, November 26 2008
Hard-to-borrow stocks: A new focus on an `old’ problem?

- Borrowing stocks is necessary for short-selling (delivery in T+2 !)
- Availability of stocks for borrowing is often limited and is variable
- Restrictions on short-selling vary strongly in time

- **Adam’s Not-so-Invisible Hand**: In September 2008 the SEC restricted for 1 month shorting in ~800 stocks (mostly financials)

- Regulation SHO: stocks must be `located’ before they are shorted

- Option market-makers are generally exempt from SHO but are subjected to **buy-ins** by their clearing brokers

- New solutions needed to achieve more transparency in stock-lending
Characteristics of HTB stocks

- Nominal Put-Call Parity does not hold (what to do about classical RN pricing?)
- Increased volatility
- Unusual pricing of vertical spreads (Put spreads/ call spreads)
- Short Squeezes
- Financing costs imply reduced, even negative, rates for shorting
## Bye-bye, Put-Call Parity

<table>
<thead>
<tr>
<th>Date</th>
<th>EXP</th>
<th>Calls</th>
<th>Puts</th>
<th>(d_{htb} = \frac{p_{pop} - c_{pop} + r_{Kt} - d_{St}}{S_t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/8/20</td>
<td>0</td>
<td>DNDN</td>
<td>3.55</td>
<td>1/2/2001 9</td>
</tr>
<tr>
<td>1/1/2000</td>
<td>9</td>
<td></td>
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<table>
<thead>
<tr>
<th>Strike</th>
<th>IVOL</th>
<th>IVOL</th>
<th>Days</th>
<th>Pmbbo</th>
<th>Cmbbo</th>
<th>Ppop</th>
<th>Cpop</th>
<th>(d_{htb})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td>Pmbbo</td>
<td>Cmbbo</td>
<td>Ppop</td>
<td>Cpop</td>
<td>(d_{htb})</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.57</td>
<td>0.58</td>
<td>121%</td>
<td>373</td>
<td>0.315</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.65</td>
<td>2.67</td>
<td>162%</td>
<td>373</td>
<td>2.29</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.8</td>
<td>2.56</td>
<td>172%</td>
<td>373</td>
<td>2.56</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>2.16</td>
<td>2.735</td>
<td>173%</td>
<td>373</td>
<td>2.135</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.45</td>
<td>1.14</td>
<td>169%</td>
<td>373</td>
<td>1.515</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.05</td>
<td>1.126</td>
<td>166%</td>
<td>373</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.86</td>
<td>1.24%</td>
<td>151%</td>
<td>373</td>
<td>1.4</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>0.7</td>
<td>1.24%</td>
<td>147%</td>
<td>373</td>
<td>0.94</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td>0.47</td>
<td>116%</td>
<td>143%</td>
<td>373</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3</td>
<td>111%</td>
<td>139%</td>
<td>373</td>
<td>0.525</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.15</td>
<td>105%</td>
<td>140%</td>
<td>373</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.11</td>
<td>102%</td>
<td>9999%</td>
<td>373</td>
<td>0.145</td>
</tr>
</tbody>
</table>
Dendreon (DNDN)

<table>
<thead>
<tr>
<th>Dendreon Corp.</th>
<th>$ 4.51</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNDN</td>
<td>-0.14</td>
</tr>
</tbody>
</table>

| Short Interest (Shares Short) | 24,337,600 |
| Days To Cover (Short Interest Ratio) | 18.6 |
| Short Percent of Float | 27.29 % |
| Short Interest - Prior | 25,076,900 |
| Short % Increase / Decrease | -2.95 % |
| Short Squeeze Ranking™ | -104 |

Short rate in October 2008 = 19.7%!
Krispy Kreme Donuts (KKD)

From 2001 to 2004, Krispy Kreme was extremely hard to borrow, with frequent buy-ins. The candlesticks show the stock was very volatile and high-priced reaching $200 (unadjusted).
VMWare Nov 07 – Sep 08
VMWare Short Rate (1/2007-8/2008)

If you short, you don’t receive interest on cash. Instead, you pay up to 25%
IOC Traded Volume
DNDN = $5.40
Jan 09 2.5P = $0.45
Jan 09 5.0P = $2.55
2.5/5.0 Put Spread = $2.10

Initial Credit = $2.10
Max Gain= $2.10 (stock>5.0)
Max Loss= $0.40 (stock<2.5)

Unusual Verticals -- Valuation in July 2008

Potential loss

Premium capture

S=5.40
Volkswagen AG
Porsche’s Short Squeeze

Porsche Long Calls; HF short stocks
# Citicorp (C)

<table>
<thead>
<tr>
<th>Financial Metric</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Citigroup Inc.</strong></td>
<td>$3.77</td>
</tr>
<tr>
<td><strong>C</strong></td>
<td>-0.94</td>
</tr>
<tr>
<td>Short Interest (Shares Short)</td>
<td>138,025,500</td>
</tr>
<tr>
<td>Days To Cover (Short Interest Ratio)</td>
<td>1.0</td>
</tr>
<tr>
<td>Short Percent of Float</td>
<td>2.70%</td>
</tr>
<tr>
<td>Short Interest - Prior</td>
<td>116,765,900</td>
</tr>
<tr>
<td>Short % Increase / Decrease</td>
<td>18.21%</td>
</tr>
<tr>
<td>Short Squeeze Ranking™</td>
<td>-2</td>
</tr>
</tbody>
</table>

**October 2008 borrow rate = -5.6%**
# Goldman Sachs (GS)

<table>
<thead>
<tr>
<th>The Goldman Sachs Group Inc.</th>
<th>$53.31</th>
</tr>
</thead>
<tbody>
<tr>
<td>GS</td>
<td>1.31</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Short Interest (Shares Short)</strong></th>
<th>9,027,400</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Days To Cover (Short Interest Ratio)</strong></td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Short Percent of Float</strong></td>
<td>2.50 %</td>
</tr>
<tr>
<td><strong>Short Interest - Prior</strong></td>
<td>7,970,200</td>
</tr>
<tr>
<td><strong>Short % Increase / Decrease</strong></td>
<td>13.26 %</td>
</tr>
<tr>
<td><strong>Short Squeeze Ranking™</strong></td>
<td>-1</td>
</tr>
</tbody>
</table>

October 2008 borrow rate = -0.1 %
General Motors Corp. (GM)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>General Motors Corp.</td>
<td>$ 3.06</td>
</tr>
<tr>
<td>GM</td>
<td>0.18</td>
</tr>
<tr>
<td>Short Interest (Shares Short)</td>
<td>102,575,700</td>
</tr>
<tr>
<td>Days To Cover (Short Interest Ratio)</td>
<td>4.3</td>
</tr>
<tr>
<td>Short Percent of Float</td>
<td>18.10 %</td>
</tr>
<tr>
<td>Short Interest - Prior</td>
<td>93,598,400</td>
</tr>
<tr>
<td>Short % Increase / Decrease</td>
<td>9.59 %</td>
</tr>
<tr>
<td>Short Squeeze Ranking™</td>
<td>-70</td>
</tr>
</tbody>
</table>
Buy-ins: A Market-Maker’s Perspective

Buy-ins Finished, Price drops

Buy-in period, Upward price pressure

9:30 AM

3 Pm (ish)

4PM close

Stock price
The Model: series of buy-ins with stochastic buy-in rate

\[
\frac{dS}{S} = \sigma dZ + \gamma \lambda dt - \gamma dN_{\lambda}(t)
\]

\[
d \ln \lambda = \kappa dW + \alpha (\ln \lambda - \ln \lambda) dt + \beta \frac{dS}{S}
\]

\[\lambda = \text{Buy-in rate} \quad N_{\lambda}(t) = \text{Poisson counter, intensity } \lambda\]
\[\gamma = \text{Scale parameter}\]
\[\beta = \text{Coupling constant}\]
\[\alpha = \text{Mean-reversion rate}\]
``Solution” of the Model

\[
d \ln \lambda = \tilde{\omega} dZ' + \alpha (\ln \bar{\lambda} - \ln \lambda) dt + \beta \gamma (\lambda dt - dN_{\lambda}(t))
\]

\[
S_t = S_0 M_t e^{\int_0^t \gamma \lambda_s ds} (1 - \gamma) \int_0^t \gamma dN_{\lambda_s}(s)
\]

\[
M_t = e^{\sigma W_t - \frac{\sigma^2 t}{2}}
\]

It can be shown that the price is a "local martingale".
Initial Lambda=20, effective dividend=10%, Gamma=0.05

Kappa=1%, Beta=5, alpha (mean rev for BIR)=1yr
Initial Lambda=2, effective dividend=1%, Gamma=0.05

Bursting behavior due to sporadic increase in buy-in rate.
Implications for option pricing

Expected loss per share due to the lack of deltas after buy-in:

\[
loss = \begin{cases} 
\gamma S, & \text{with prob } \lambda dt \\
0, & \text{with prob } 1 - \lambda dt 
\end{cases}
\]

\[
E(loss \mid \lambda, S) = \gamma \lambda S dt
\]

Agents would pay this to ensure that the short stock is not removed.

Conclusion: there is a convenience yield for holding stock (you can lend it) and the "fair dividend rate" (stochastic) is

\[
d_t = \gamma \lambda_t
\]
Risk-neutral measure

\[
\frac{dS}{S} = \sigma dW + \gamma(\lambda dt - dN_\lambda(t)) - \gamma \lambda dt + rdt
\]

\[
= \sigma dW - \gamma dN_\lambda(t) + rdt
\]

\[
S_t = S_0 e^{\sigma W_t - \sigma^2 t / 2 + rt} \cdot (1 - \gamma) \int_0^t dN_\lambda(s)
\]

\[
E(S_t) = S_0 e^{rt} E \left\{ e^{-\gamma \int_0^t \lambda_s ds} \right\}
\]
Implications for Option Pricing

Forward Price \( (T) = S_0 e^{rT} E \left\{ e^{-\gamma \int_0^T \lambda_s ds} \right\} \)

Put\( (S, K, T) \) – Call\( (S, K, T) = K \cdot e^{-rT} - S \cdot E \left\{ e^{-\gamma \int_0^T \lambda_s ds} \right\} \)

\( d_{eff}(T) = -\frac{1}{T} \ln E \left\{ e^{-\gamma \int_0^T \lambda_s ds} \right\} \)  

Term structure of div rates
Implied Dividend Rate

\[(ATM \text{ Put}) - (ATM \text{ Call}) = Ke^{-rT} - Se^{-D_{imp}(T)T}\]

\[D_{imp}(T) = -\frac{1}{T}\ln\left(\frac{(ATM \text{ Put}) - (ATM \text{ Call}) - Ke^{-rT}}{S}\right)\]

The model gives a term-structure of effective dividends based on the anticipations for hard-to-borrowness (specialness) of the stock in the future.
Term-structure of implied dividends
Option Pricing

European options

Define: \( \Pi(n,T) = \Pr \left\{ \int_0^T dN_{\lambda_t}(t) = n \right\} \)

\[
= E \left\{ \frac{\left( \int_0^T \lambda_t dt \right)^n}{n!} e^{-\int_0^T \lambda_t dt} \right\}
\]

\[
Call(K,T) = \sum_{n=0}^{\infty} \Pi(n,T) \cdot BSCall(S(1-\gamma)^n, T, K, r, \sigma)
\]
Poisson Weights for Option Pricing
American Options

- Need a 2D lattice method, or LSMC, to price
- Sometimes, it is optimal to exercise American Calls (even if there are no actual dividend payments)
- Deep ITM calls need to be hedged with a lot of short stock. The advantages of holding a synthetic puts may be compensated by the risk of buy-ins or negative carry for holding stock
- This is clearly confirmed by market observations
Dendreon Corp. (DNDN) Jan 09

The diagram shows the relationship between the effective dividend and the strike price. The x-axis represents the strike price in dollars, and the y-axis represents the effective dividend. The graph indicates the call early exercise and put early exercise scenarios.
VMWare: Implied Dividend
Theoretical Implied Dividend
(Calls have early exercise)
Extracting HTB Value from Spread Trades with Leveraged ETFs

\[ S_t : \text{IYF, I - Shares Dow Jones U.S. Financials} \]
\[ L_t^+ : \text{UYG, double long financial ETF} \]
\[ L_t^- : \text{SKF, double short financial ETF} \]

Managers of short-leveraged ETFs must borrow shares of the underlying index, incurring an additional cost.

This cost is stochastic, it depends on how difficult it is to borrow the underlying securities.
Leveraged ETF Spreads

\[
\frac{dL_t^+}{L_t^+} = 2 \frac{dS_t}{S_t} - rdt - fdt
\]

\[
\frac{dL_t^-}{L_t^-} = -2 \frac{dS_t}{S_t} + rdt + 2(r - \delta \lambda_t)dt - fdt
\]

\[
\frac{dL_t^+}{L_t^+} + \frac{dL_t^-}{L_t^-} = 2(r - f)dt - 2\delta \lambda_t dt
\]

\[
\delta \lambda_t dt = -\frac{1}{2} \left( \frac{dL_t^+}{L_t^+} + \frac{dL_t^-}{L_t^-} \right) + (r - f)dt
\]

Short-short position in UYG and SKF pays the hard-to-borrowness
Evolution of the short SKF, short UYG spread, Feb 2007- Feb 2009
Conclusions

- HTB stocks and their options are interesting!
- Options on HTBs present breakdown of nominal Put-Call Parity
- Puts and Calls are`` in equilibrium”, but we must anticipate the cost of carry, or convenience yield
- Introduced a model for the fluctuations of prices based on an additional factor, the buy-in rate
- Model explains the term-structure of implied dividends
- American calls have optimal early exercise
- Generalization to **impossible to short stocks** (China, HK. Financials in Sep 2008)