1. **Currency ETF pairs.** Consider the ETFs (i) Wisdom Tree Dreyfus Brazilian Real (BZF) and (ii) CurrencyShares Euro Trust (FXE).

(a) An investor implements a strategy whereby he or she buys one million (1mm) USD of BZF on June 6, 2008 and holds the position until March 30 2010. Evaluate the return, Sharpe ratio and maximum drawdown, defined as \( \max_t (V(t) - \min_{s \geq t} V(s)) \).

(b) Analyze another strategy whereby the investor holds every day 1mm USD long BZF and 1mm USD short FXE over the same period. Assuming that the account has an initial equity of 1mm, analyze the return, Sharpe ratio and maximum drawdown.

(c) A third investor (who took Math Finance at the Courant Institute) implements a dynamic strategy: she observes the process associated with the long-short spread in (b) and calculates its 20-day moving average and standard deviation (looking back daily for 20 days, beginning on 6/26/2008). She builds the following processes:

\[
X_n = X_0 + \sum_{j=1}^{n} (R_{BZF,j} - R_{FXE,j})
\]

\[
\mu_n = \frac{1}{20} \sum_{k=1}^{20} X_{n-k}
\]

\[
\sigma_n^2 = \frac{1}{20} \sum_{k=1}^{20} (X_{n-k} - \mu_n)^2
\]

\[
Y_n = X_n - \mu_n
\]

Interpret \( X_n \) and \( Y_n \) in words. The trader implements the strategy of buying the BZF/FXE spread when \( Y_n < -2.0\sigma_n \) and shorting the spread when \( Y_n > +2.0\sigma_n \). Backtest this strategy from June 26, 2008 until March 30 2010, calculating return, Sharpe and maximum drawdown.

2. **ARMA & GARCH.** Consider the time-series \( X_n \) defined in the previous problem. Fit \( X_n \) to an autorregresive ARMA(1,1) model with GARCH(1,1) volatility. Use all the data in the sample for the estimation. Derive, if possible, an ex-post improved estimation window for the dynamic strategy described in Problem 1 [Hint: 20 days might be too short.]

3. **PCA on Implied Volatilities for Equity Options.** Using the panel data in the attached spreadsheet, perform a sequential, PCA study of the 30-day at-the-money implied volatilities for the components of Nasdaq 100 Index between Sept 4, 2007 and Oct 30, 2009. Suggested window size: 120 business days. Characterize the top eigenvector of the correlation matrix and analyze its stability in time. compare the returns of the corresponding “eigenportfolio” with (a) QQV (Nasdaq Volatility Index) (b) an equal-weighted average of volatilities. What can you say about the other “significant” eigenvectors obtained by PCA?