Complex Variables II PROBLEM SET 2 Due February 6, 2007

1-5. Problems 4, 6, 7,8, 10 pages 370-372 of text.

6.* (a) What is the image of the interior R of the half-disk

$$|z| < 1,\ Im(z) > 0,$$

under the mapping $Z = -\frac{1}{2}\big(z + 1/z\big)$?

Indicate the images of the points A, B, C, D, E and the interior of R with a sketch of the Z-plane.

(b) Find the harmonic function $T(x, y)$ defined in R which satisfies

$$T = 1 \ on \ \{z, |z| = 1, 0 < arg(z) < \pi\}$$

and

$$T = 0 \ on \ \{z = x + iy : y = 0, -1 < x < +1\}.$$

Express T as an explicit function of (x, y) and verify that the required boundary conditions are satisfied.