1. Problem 2, page 153 of text. (Recall, if \( f \) is analytic within the region bounded by two simple, closed, positively oriented contours, and is also analytic on the contours, then the integral of \( f \) is the same on each contour.)

2. Evaluate

\[
I(z_0) = \int_C \frac{dz}{z - z_0}
\]

taken once around the contour shown in the figure, for all point \( z_0 \) not on the contour. Be sure to consider \( z_0 \) located in each of the regions I, II, III, IV.


4. If \( P(z) \) is the polynomial \( \Pi_{k=1}^n (z - z_k) \) where the \( z_k \) are complex numbers, and \( C \) is a simple closed contour containing all of the \( z_k \), show that

\[
\int_C \frac{P'(z)}{P(z)} \, dz = 2\pi in.
\]

5. By using the method and the rectangular contour of problem (3) above, applied now to the function \( f(z) = \frac{1}{1 + z^2} \), show that

\[
\int_{-\infty}^{+\infty} \frac{(1 - b^2 + x^2)}{(1 - b^2 + x^2)^2 + 4b^2 x^2} \, dx = \pi,
\]

where \( b \) is real and \( 0 < b < 1 \). (Recall \( \int \frac{1}{1+x^2} \, dx = \tan^{-1} x \). Be sure to recognize integrals of functions which vanish by being odd in \( x \). Also note that \( |1 + a^2 - y^2 + 2iay| \geq |1 + a^2 - y^2| \geq 1 + a^2 - b^2 > 0 \) on the integrals on the vertical sides.)

6. Evaluate

\[
\int_0^\infty \frac{\sin x}{\sqrt{x}} \, dx
\]

By integrating \( \frac{\sin z}{\sqrt{z}} \) over the contour \( C = C_1 + C_2 + C_3 + C_4 \) shown in the figure, involving arcs of radius \( \epsilon \) and \( R \), then letting \( \epsilon \to 0 \) and \( R \to \infty \).(Hint: To estimate the integral \( C_2 \) use \( \sin \theta \geq 2\theta / \pi, 0 \leq \theta \leq \pi / 2 \). You should show that \( C_4 \) vanishes as \( \epsilon \to 0 \) and that \( C_2 \) vanishes as \( R \to \infty \). Take \( \sqrt{z} = \sqrt{\epsilon} e^{i\theta/2}, 0 \leq \theta \leq \pi / 2 \). You can check your result against \( \int_0^\infty \sin(t^2) \, dt \) as done in class by a change of variables.)