1. Problem 2, page 163 of text.

2. If the points $z = a$ and $z = b$ are inside the domain bounded by a simple closed contour $C$, show that
\[ \int_C e^{(z-a)(z-b)} \, dz = 0. \]

3. We showed in class the the Legendre polynomial $P_n(z)$ has the representation
\[ P_n(z) = \frac{1}{2\pi i} \oint_C \frac{(s^2 - 1)^n}{2^n(s - z)^{n+1}} \, ds. \]
Here $z$ lies within the positively oriented contour $C$. Show, by taking $C$ to be of center $z$ and radius $\sqrt{|z^2 - 1|}$, that
\[ P_n(z) = \frac{1}{\pi} \int_0^{\pi} (z + \sqrt{z^2 - 1} \cos \theta)^n \, d\theta. \]
This is a formula given by Laplace. (Hint: Write $s = z + \rho e^{i\phi}$, $0 \leq \phi \leq 2\pi$. Also let $\sqrt{z^2 - 1} = \rho e^{i\alpha}$. Then arrange the integrand so that $\theta = \phi - \alpha$ is the parameter of the circular contour.)

4. Problem 1 page 171 of text.

5. Problem 4, page 172 of text.

6.** Let $f(z)$ be an entire function (i.e. analytic in the entire complex plane), and let $M(R) = \max_{|z|=R} |f(z)|$, for $R > 0$. Suppose that $M(2R) \leq 2^n M(R)$ for all $R > 0$, and for some positive integer $n$. Show that then $f(z)$ is a polynomial of degree not exceeding $n$. (Note: ** means this is a written comprehensive examination question just as presented there, with no hints. Try to solve it, but if you have problems go to the course web site to find a hint to one way to do it.)