Applied Mathematics II **PROBLEM SET 1** **Due January 30, 2001**

1. (10pts)This is problem 3 on page 92 of G & L: Find the solution of $u_{tt} - c^2 u_{xx} = 0$ when

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ 2 - x, & 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}, \; g(x) = 0.$$

Assuming $c = 1$, sketch $u(x,0), u(x,1), u(x,2)$.

2. (10pts)Consider the solution of $u_{tt} - c^2 u_{xx} = 0$ in $x \geq 0, t \geq 0$ satisfying $u(x,0) = 0, u_t(x,0) = 0, u(0,t) = h(t)$, h a twice continuously differentiable function satisfying $h(0) = 0$.
a) $u(x,t)$ will then vanish for $0 \leq t \leq x/c$. Why?
b) What is the solution of this problem, in terms of h?

3. (30pts)This is problem 7 on page 93 of G & L, (but note I have used primes for dummy variables rather than follow the notation of the text). Consider the solution of the inhomogeneous wave equation $u_{tt} - c^2 u_{xx} = F(x,t)$ in the entire $x - t$ plane, with $F(x,t) = 0, t < 0$.
a) Introduce *characteristic coordinates* $\alpha = x + ct, \beta = x - ct$. Show that is these coordinates the problem may be written in the $\alpha - \beta$ plane as

$$\frac{\partial^2 u}{\partial\alpha\partial\beta} \equiv u_{\alpha\beta} = \frac{-1}{4c^2} F(\frac{\alpha+\beta}{2}, \frac{\alpha-\beta}{2c}).$$

b) By integration, show that for integrable F we have

$$u(\alpha,\beta) = \frac{1}{4c^2} \int_\beta^\infty \int_{-\infty}^\alpha F(\frac{\alpha'+\beta'}{2}, \frac{\alpha'-\beta'}{2c}) d\alpha' d\beta'.$$

c) Change variable back to x, t in the last solution and show that

$$u(x,t) = \frac{1}{2c} \int_0^t \int_{x-c(t-t')}^{x+c(t-t')} F(x',t') dx' dt'.$$

Refer to figure 4.3 of G& L. Show that this can be written

$$u(x,t) = \frac{1}{2c} \int \int_\Delta F(x',t') dx' dt',$$

where Δ is the triangle of the $x - t$ plane formed by the characteristics through (x,t) and the x-axis.
d) Verify by differentiation that this solves the inhomogeneous equation we started with.

4. (10 pts) This is problem 8, page 84 of G& L. Using the result of the last problem, find the D'Alembert form of the solution of the inhomogeneous IVP: $u_{tt} - c^2 u_{xx} = F(x,t)$, $u(x,0) = f(x), u_t(x,0) = g(x)$, by adding the solutions of two problems.

5. (20 pts) This is problem 2, page 99 of G& L: solve formally by separation of variables

$$u_{tt} - c^2 u_{xx} = 0, 0 < x < L, t > 0$$

$$u_x(0,t) = u_x(L,t) = 0, t \geq 0$$

$$u(x,0) = f(x), u_t(x,0) = g(x), 0 \leq x \leq L.$$