

1. Problem 1.4, page 34 of text.

1. Problem 1.4, page 34 of text. Give the integral curves in parametric form with parameter  $t$ , so that the curve goes through the point  $(a, b, c)$  when  $t = 0$ . Show in part (b), for example, that the curves are given by

$$x = t + a, y = \pm[b^4 + (a^4 - (t + a)^4)]^{1/4}, z = c$$

2. Problem 2.7 page 41 of text. Again give the integral curves in parametric form as in problem 1.  
3. Solve the following problems for  $u(x, y)$ . Be sure to eliminate  $a, b, c, t$  to express  $u$  as a function of  $x, y$ . In each case check your answer by partial differentiation:

$$(a) \quad xu_x + u_y = 1, u(x, 0) = e^x.$$

$$(b) \quad xu_x + (y^2 + 1)u_y = u, u(x, 0) = e^x.$$

4. Solve the linear problem

$$u_t - tx^2u_x = 0, u(x, 0) = x + 1,$$

and sketch the characteristic curves in the  $x, t$ -plane. Show that the solution becomes infinite on a certain curve in the  $x, t$  plane.

5. Solve the linear first-order equation

$$u_t + e^y u_x + u_y = 0, u(x, y, 0) = x + y.$$

First find the characteristic curves in the  $x, y$ -plane and give the functions  $x(a, b, t)$  and  $y(a, b, t)$  which determine these curves. Then find  $u(x, y, t)$ .