INVITED LECTURES ON GAME THEORY BY SYLVAIN SORIN

SYLVAIN SORIN, MARCH 2017

Professor Sylvain Sorin (Université Pierre et Marie Curie - Paris 6) will visit NYU and deliver a series of lectures on game theory in March 2017. These lectures, of interest to mathematicians, economists and computer scientists, will cover equilibria in games with a focus on dynamic games and learning in games. A renowned expert on game theory, Sylvain Sorin made important contributions to these topics. He is a past editor-in-chief of the *International Journal of Game Theory*, the author of over 70 research papers, six books including more recently the monograph of the Econometric Society *Repeated Games* (co-authored with Jean-Francois Mertens and Shmuel Zamir).

Practical information

Three topics will be covered: finite composite games, asymptotic values of dynamic games, and strategic learning. The lectures will take place in the Courant building (251 Mercer Street), at the following times and places:

- Wednesday, March 22, 9:00 to 10:50am WWH 517 (Finite Composite Games)
- Thursday, March 23, 9:00 to 10:50am WWH 1302 (Asymptotic value of dynamic games)
- Wednesday, March 29, 9:00 to 10:50am WWH 517 (Strategic learning)
- Thursday, March 30, 9:00 to 10:50am WWH 1302 (More on one of the three topics above)

1. Finite composite games: equilibria and dynamics

We define finite games as having finitely many participants and choices, with the following types of participants:
(I) population of nonatomic players,
(II) atomic splittable players,
(III) atomic non splittable players.
We recall conditions allowing to express equilibria through variational inequalities. We introduce then potential games and dissipative games, as well as some associate evolutionary dynamics (replicator, Smith, BvNN, local and global projection, best reply).
Then we consider composite games where the 3 types of participants coexist, a typical example being congestion games, and extend the previous properties of equilibria and dynamics.
We will also describe properties of the equilibrium correspondence in this framework and discuss extensions to the continuous choice case.

References

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2. Asymptotic value of dynamic games

Long term strategic interactions in a stationary environment have been usually modelized as repeated games.

At each stage n of the process, the moves of the players and the current state determine a payoff g_n and the joint law of the new state and signals to the players. An evaluation $\theta = \{\theta_n; \theta_n \geq 0, \sum_{n\geq 1} \theta_n = 1\}$ induces a total payoff $\sum_n \theta_n g_n$, hence a game, with value v_θ.

Longer games, when the duration associated to θ increases, correspond to *vanishing stage weight* and the associated limit of the values $\{v_\theta\}$ is *the asymptotic value*.
We will describe recent advances involving new approaches and results (including cases of existence and non-existence of the limit).

An alternative approach for studying repeated interactions considers a continuous time process on $\mathbb{R}^+$ that the players control at discrete times, corresponding to a partition Π of $\mathbb{R}^+$. The asymptotic approach is the analysis of the game as the mesh of Π decreases, thus with *vanishing stage duration*.
We will present new developments in this direction and discuss the relation with differential games or more generally games in continuous time.

In both frameworks the main tool is the recursive structure of the game and the associate operator that extends the initial Shapley formula for finite discounted stochastic games.

References

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3. Topics on strategic learning

This introductory lecture will present models and algorithms related to learning in a strategic framework.

A first part will cover unilateral procedures where an agent faces an unknown environment. The model is in discrete time and the agent chooses at each stage an action based on his previous knowledge. He observes then an outcome and gets a reward. His objective is to maximize the flow of such rewards.
We will present the basic tool : "approachability theory" and the induced algorithms for "no-regret" or "calibration" criteria.
We will then describe some extensions and recent results including relation with on-line optimization and the imperfect monitoring case.

A second part will consider global dynamics where all the players in a game use a specific learning procedure. We will describe various convergence results for some classes of games and types of equilibria: internal/external consistency and Hannan/correlated equilibria, fictitious play and Nash equilibria.

Finally we will deal with procedures in continuous time and prove some convergence properties. Then we will present results allowing to deduce convergence in the original discrete time configuration. This applies in particular to smooth fictitious play, exponential weight algorithm and mirror descent approaches.

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