Continuous Time Finance, Spring 2004 – Homework 5
Posted 4/3/04, due 4/14/04 (TYPO IN THE SDE FOR C CORRECTED 4/7/04)

In problem 3 of HW1 we considered options on a foreign exchange rate, assuming the interest rate in each currency was constant. Now we have more sophisticated interest rate models; let's see how they work in this setting. Let $C(t)$ be the exchange rate in dollars/pound, and consider an option that gives a dollar investor the right to buy pounds at exchange rate K at time T; its payoff (to the dollar investor) is

$$(C(T) - K)_+ \quad \text{dollars at time } T. \tag{1}$$

Use subscripts D, P, and C to distinguish analogous dollar, pound, and exchange-rate objects: for example

$P_D(t,T) =$ dollar value at time t of a zero-coupon bond worth one dollar at time T.

Use Hull-White models for the dollar and pound short rates:

$$dr_D = (\theta_D(t) - a_D r_D)\,dt + \sigma_D dw_D$$

where w_D is a Brownian motion under the dollar investor's risk-neutral measure; and

$$dr_P = (\theta_P(t) - a_P r_P)\,dt + \sigma_P dw_P$$

where w_P is a Brownian motion under the pound investor's risk-neutral measure. Assume the exchange rate has constant drift and volatility:

$$dC = \mu_C C\,dt + \sigma_C C dw_C$$

where w_C is a Brownian motion under some (subjective) probability. The Brownian motions may be correlated: assume

$$dw_D\,dw_P = \rho_{DP}\,dt, \quad dw_D\,dw_C = \rho_{DC}\,dt, \quad dw_P\,dw_C = \rho_{PC}\,dt,$$

where ρ_{DP}, ρ_{DC}, and ρ_{PC} are constant.

(a) What is the value (to the dollar investor, at time $t < T$) of the payoff (1)? (Make your answer as explicit as possible.)

(b) Describe a trading strategy for the dollar investor that replicates this payoff. (Again, be as explicit as possible.)

(c) Is a similar analysis possible if we use one-factor HJM models for the interest rates rather than Hull-White?

[Extra credit: consider the analogous question for quanto call, whose value to the dollar investor is $(S(T) - K)_+$ at time T, where S is the price of a stock in pounds. This is of course the stochastic-interest-rate analogue of our discussion of quantos, in Section 3 and problem 4 of HW2.]