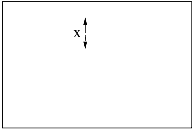
A game

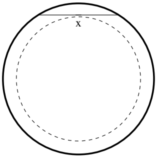
Paul wants to exit; Carol wants to stop him; step size is $\varepsilon > 0$



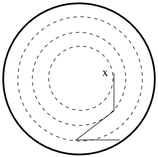
- Paul chooses a direction
- Carol may reverse it
- Paul moves distance ε

Can Paul exit? How?

Circles are easier than rectangles



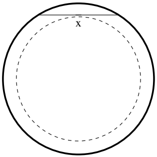
If region is a circle, Paul can exit in one step from a slightly smaller circle.



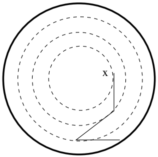
For a circle, the sets from which Paul can exit in *j* steps are concentric circles.

For any convex domain, Paul's optimal strategy can be identified similarly. The set from which he can exit in one step is traced by the midpoints of secants of length 2ε.

Circles are easier than rectangles

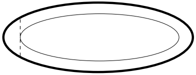


If region is a circle, Paul can exit in one step from a slightly smaller circle.



For a circle, the sets from which Paul can exit in j steps are concentric circles.

For any convex domain, Paul's optimal strategy can be identified similarly. The set from which he can exit in one step is traced by the midpoints of secants of length 2ε.



A simpler strategy

Suppose Paul is lazy – he wants a simple strategy that always works.

Suggestion: fixing an origin inside the domain, Paul can use the strategy that's optimal for circles:

If Paul always chooses the direction tangent to the circle, then by Pythagoras' theorem

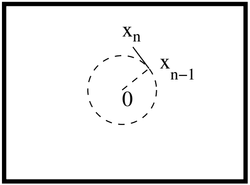
$$|x_n|^2 = |x_{n-1}|^2 + \varepsilon^2.$$

By this strategy he makes steady progress away from 0, so he eventually leaves any bounded set.

A simpler strategy

Suppose Paul is lazy – he wants a simple strategy that always works.

Suggestion: fixing an origin inside the domain, Paul can use the strategy that's optimal for circles:



If Paul always chooses the direction tangent to the circle, then by Pythagoras' theorem

$$|x_n|^2 = |x_{n-1}|^2 + \varepsilon^2.$$

By this strategy he makes steady progress away from 0, so he eventually leaves any bounded set.

Some comments

- Two-person games arise a lot in economics. Optimal decision-making is the focus of operations research.
- The Paul-Carol game involves geometry. As $\varepsilon \to 0$ Paul's optimal strategies are described by partial differential equations. I studied this with Sylvia Serfaty (Comm Pure Appl Math 59, 2006, 344-407).
- For a brief expository discussion see my article *Parabolic PDE's and deterministic games* in SIAM News, Oct 2007 https://www.siam.org/pdf/news/1208.pdf

Enough games for now

Today's main feature is Professor Chris Budd, from University of Bath, telling us about

Vital Math:
How mathematicians changed the world