MECHANICS
MATH-GA.2710-001, Spring 2018
Wednesdays 1:25-3:15pm, WWH 705

Version 4, 2/21/2018: the 3/21 class is NOT cancelled

Instructor: Robert Kohn (kohn@cims.nyu.edu, 8-3217, WWH 502). Office hours: Tues 2-3 and Wed 3:30-4:30 (or by appt).

Description: This course provides brief introductions to elasticity, classical mechanics, and statistical mechanics – topics at the interface where ODE, PDE, and probability meet physics and materials science. For students preparing to do research on physical (or biophysical) applications, the class provides an introduction to crucial concepts and tools; for students of analysis the class provides valuable context by exploring some applications. No prior exposure to mechanics or physics is assumed. The scope of the material to be discussed is vast, so we must make some choices. We'll spend roughly half the semester on elasticity, and a quarter semester each on classical mechanics & statistical mechanics:

- Our discussion of *elasticity* (about 7 weeks) will start with one-dimensional models (strings and rods) and buckling (as an example of bifurcation); then we'll continue with finite elasticity and linear elasticity in the context of 3D solids.
- Our discussion of classical mechanics (about 3 weeks) will start with Hamilton's equations and action minimization as alternative formulations; then we'll explore connections with PDE and the calculus of variations (including geodesics and optimal control).
- Our discussion of statistical mechanics (about 3 weeks) will start with basic concepts such as the microcanonical and canonical ensembles, entropy, and the equilibrium distribution; then we'll continue with the Ising model (as an example of phase transition) and Metropolis sampling (as an example of a Markov Chain Monte Carlo method).

One viewpoint on this class is that it complements the Fall semester Fluid Dynamics course, by providing an introduction to other areas of mechanics. (Please note, however, that Fluid Dynamics is *not* a prerequisite for this class.)

Calendar: The first lecture is Wed 1/24. There will be no class on Wed 3/14 (spring break). The last lecture is Wed 5/2. The finals-week slot (Wed 5/9) will be used for student presentations.

Prerequisites: The specific prerequisites are ODE and PDE at the level of an undergraduate class (or preferably, in the case of PDE, at the level of PDE-I). However this course also demands substantial mathematical maturity: while each segment starts at the beginning, we will move quickly. The course has been designed with first and second year PhD students in mind; Masters students should consult with the instructor before registering.

Requirements: The course requirements are two-fold: (1) problem sets, and (2) a 25-minute presentation. The problem sets (assigned after Lectures 1, 3, 5, and 7, normally due two weeks after distribution) will focus on the Elasticity segment of the class; students are encouraged to discuss the problems, but are expected to write up their solutions individually. The 25-minute presentations can be on any topic within the course's scope; some suggestions related to our elasticity segment will be distributed around spring break. Students should choose their presentation topics by the beginning of April. We will use the May 9 class slot for presentations; if (as seems likely) more time is needed, additional presentation sessions will be scheduled during the last week of classes or finals week. There will be no final exam.

Websites: I will put scanned notes and homework assignments on a public website https://math.nyu.edu/faculty/kohn/mechanics.html . (This site also has a link to my notes and HW from Spring 2012, when I last taught this class; the 2018 version will be similar, though not identical.) The course will also have an NYU Classes site, for material that's intended only for registered students (e.g. homework solutions), and for communication by email.

Books: We will not follow any text linearly. But (I hope) you'll want to know more about each topic than we can possibly cover in a few lectures. Therefore the following books might be useful (all will be on reserve in the Courant library).

SOLID MECHANICS

- P. Howell, G. Kozyreff, J. Ockendon, *Applied Solid Mechanics*, Cambridge Univ Press 2009 (available electronically through Bobcat).
- G. Duvaut and J.-L. Lions, *Inequalities in mechanics and physics*, Springer, 1976 (available electronically through Bobcat).
- S. Antman, *Nonlinear problems of elasticity*, Springer-Verlag, 2005 (available electronically through Bobcat).
- P.G. Ciarlet, *Mathematical elasticity I: Three-dimensional elasticity*, North-Holland, 1988 (available electronically through Bobcat).
- J.E. Marsden and T.J.R. Hughes, *Mathematical foundations of elasticity*, Prentice-Hall, 1983 (now available inexpensively from Dover).

Howell-Kozyreff-Ockendon provides a wide-ranging introduction at a relatively basic level; if you're new to this topic, it could be worth reading cover-to-cover. The other books are much more advanced (not recommended for cover-to-cover reading). They are in fact quite different from one another. Duvaut & Lions is very "French" (emphasizing weak solutions, for example), with a focus on physically nonlinear but geometrically linear models. Antman by contrast emphasizes geometrically nonlinear models. Ciarlet provides the basics of nonlinear elasticity in a clean, well-organized, but rather dry form. The first 25 pages of Marsden & Hughes gives a concise introduction to nonlinear elasticity.

CLASSICAL MECHANICS

- O. Buhler, *A brief introduction to classical, statistical, and quantum mechanics*, Courant Lectures Notes in Mathematics vol 13, 2006.
- V. I. Arnol'd, *Mathematical methods of classical mechanics*, Springer (available electronically through Bobcat).

- L.D. Landau and E.M. Lifshitz, *Mechanics*. (The 3rd edition is available electronically through Bobcat).
- J.V. José and E.J. Saletan, *Classical dynamics: A contemporary approach*, Cambridge University Press, 1998 (available electronically through Bobcat).

Another source – not in the CIMS library (so not on reserve and therefore not listed above) but available online through the Springer book package – is E. DiBenedetto, *Classical Mechanics: Theory and Mathematical Modeling*, Birkhauser, 2011.

Buhler's book has the advantage of conciseness, but the disadvantage of having few examples or exercises – overall: highly recommended but not sufficient. Landau & Lifshitz (a classic) and José & Saletan (more modern) have many more examples and exercises; I prefer the latter due to its more modern and geometric perspective. DiBenedetto's book seems to be at about the same level as José & Saletan. Arnol'd's book is deeper and more advanced; while it does cover material similar to ours in the first few chapters, most of the book discusses a geometric side of the subject that we won't have time to get to.

STATISTICAL MECHANICS

- O. Buhler, *A brief introduction to classical, statistical, and quantum mechanics*, Courant Lectures Notes in Mathematics vol 13, 2006.
- A. Chorin and O. Hald, *Stochastic tools in mathematics and science*, Springer, 2013 (available electronically through Bobcat).
- D. Chandler, *Introduction to modern statistical mechanics*, Oxford University Press, 1987.
- C. Thompson, *Mathematical statistical mechanics*, Princeton Univ Press, 1978 (available electronically through Bobcat).

Another source – not a book, so not on reserve and not listed above – is Veit Elser's *Three lectures on statistical mechanics*, arxiv:1612.00863.

The comment made earlier about Buhler's book applies here too. Chapter 5 of Chorin & Hald is similarly concise, with a choice of topics somewhat different from Buhler's (it includes discussions of real-space renormalization and Markov Chain Monte Carlo). For a more extensive treatment that's still at a fairly elementary level, Chandler's book is the best I know. Thompson's book is quite different from Chandler's, partly because it was written for a mathematical audience while Chandler writes for an audience of theoretically-minded chemists and physicists. Thompson's choice of topics reflects the book's age (eg Onsager's solution of the 2D Ising model is given in detail, but real-space renormalization is absent, as is Markov Chain Monte Carlo); still, it is a good source for the topics it covers. Elser's 43-page arxiv posting is, to quote the abstract, "a lightweight introduction to statistical mechanics for mathematicians." Its choice of topics is refreshingly modern (and rather different from Buhler or Chorin & Hald), making this a good complement to the other items listed.