# Introduction to PDEs 2018, fourth assignment, due Tuesday October 9

**1)** Solve the initial value problem for the Schrödinger equation

$$i\psi_t + \psi_{xx} = 0, \quad \psi(x,0) = e^{-\frac{1}{2}x^2},$$

through the following steps:

1. Find exact solutions to the equation of the form $\psi_k = e^{i(kx-\omega t)}$ for a suitable dispersion relation $\omega = \Omega(k)$.
2. Consider a general linear superposition of such solutions,
$$\psi(x,t) = \int_{-\infty}^{\infty} \hat{\psi}(k) e^{i(kx-\omega t)} dk$$
and identify the proper $\hat{\psi}(k)$ by setting $t = 0$ (For this you might need to refresh your knowledge of the Fourier transform) and completing squares.
3. Again completing squares, find a close form for $\psi(x,t)$.

**2)** Rewrite your solution $\psi(x,t)$ above in the form

$$\psi(x,t) = a(x,t)e^{i\theta(x,t)}$$

with $\theta$ a real phase, consider the asymptotic behavior of $\theta$ for large values of $t$, and identify the wave number $k(x,t)$ and $\omega(x,t)$. Verify that, for large values of $t$, $k$ is constant along paths moving at the group velocity $C_g(k)$.

**3)** Plot (using for instance Matlab) the real part of your solution $\psi(x,t)$ from part 1 at $t = 8$ for $x$ between $x = 0$ and $x = 60$. Display on the same plot a sinusoidal $y = \alpha \sin(2x)$ (with an amplitude $\alpha$ chosen so that it fits on the same plot), and verify that the wavelengths of the two plots agree where they should according to dispersive wave theory.