Assignment 3

Due March 4

Consider the upper half plane $y \geq 0$. If $f(x)$ is a function in $L_2(R)$ show that

$$u(x,y) = \frac{y}{\pi} \int \frac{f(x+z)}{z^2+y^2} dz$$

is well defined, satisfies for $y > 0$

$$u_{xx} + u_{yy} = 0$$

and

$$\lim_{y \downarrow 0} \int_R |u(x,y) - f(x)|^2 dx = 0$$

When will

$$\lim_{y \downarrow 0} \int_R |u_y(x,y) - g(x)|^2 dx = 0$$

for some $g \in L_2$. Can you express g in terms of f.