Q1. Consider the map

$$(Kf)(x) = \int_{R^3} \frac{f(y)}{|x-y|} dy$$

acting on C_∞ functions with compact support on R^3. Show that f is C_∞ and $\Delta K = cI$ i.e $\sum_i \frac{\partial^2}{\partial x_i^2}(Kf) = cf$ for some constant c and evaluate c.

Q2. What would the corresponding result be in dimension 1 and 2.

Q3. Show that it is not possible for both the function f and its Fourier transform $\widehat{f}$ to have compact support on R.

Q4. Show that if $f, \widehat{f}$ are Fourier transforms $\|f\|_2 = \|\widehat{f}\|_2 = 1$, then

$$\Big(\int x^2 |f(x)|^2 dx\Big)\Big(\int y^2 |\widehat{f}(y)|^2 dy\Big) \geq \frac{1}{4}$$

Q5. Show that every permutation group has exactly rwo one dimensional representations, the identity and the signature.

Q6. How many inequivalent representations does the permutation group of 4 elements have and what are their dimensions?