

1. If  $T : X \rightarrow X$  is  $I + F$  where  $I$  is identity and  $F$  is an operator of finite rank, show that its index is 0 by explicit calculation.

2. Let  $H = \{f : f = \sum_{k=0}^{\infty} a_k e^{ikx}\}$  where  $\sum_{k=1}^{\infty} |a_k|^2 < \infty$ .  $H \subset L_2[S]$  with  $S = \{z : |z| = 1\}$ .  $P : L_2(S) \rightarrow H$  is the orthogonal projection onto  $H$ . If  $f(s)$  is a continuous function  $S \rightarrow \mathbb{C}$  with  $f(s) \neq 0$  for any  $s \in S$  then show that  $g \rightarrow P \frac{1}{f} P f g$  is Fredholm and calculate its index. The map  $T_f : H \rightarrow H$  is multiplication of  $g \in H$ , i.e. a function with nonnegative Fourier coefficients by a smooth function  $f$  projecting out the negative frequencies, multiplying by  $\frac{1}{f}$  and projecting out again the negative frequencies.