Let $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent random variables such that $E[X_i] = 0$ for $i = 1, 2, \ldots$. However they are not assumed to have the same distribution. We are interested in proving the weak law of large numbers, i.e. that

\[ \lim_{n \to \infty} P \left( \left| \frac{X_1 + X_2 + \cdots + X_n}{n} \right| \geq \epsilon \right) = 0 \]

for every $\epsilon > 0$.

1. Show that the weak law of large numbers is not true in this generality by constructing a counterexample.

2. Show that it does not help even if we make the extra assumption that

\[ \sup_n E[|X_n|] < \infty \]

3. Show that if we make the assumption that

\[ \sup_n E[|X_n|^{1+\delta}] < \infty \]

for some $\delta > 0$, then the weak law holds.