Limit Theorems.

Set 6. Due Nov 21, 2002.

Q1. Let $X_{n,1}, X_{n,2}, \ldots, X_{n,n}$ be a triangular scheme of mutually independent uniformly infinitesimal random variables. We are interested in the limiting distribution of $Y_n = X_{n,1} + X_{n,2} + \cdots + X_{n,n}$. Assume that Y_n has a limiting distribution α . Show that α is Gaussian if and only if

$$\lim_{n\to\infty} \sum_{j=1}^{n} P[|X_{n,j}| \geq \epsilon] = 0$$

for every $\epsilon > 0$.

Q2. Can you give an example of a triangular scheme $\{X_{n,j}\}$ such that $E[X_{n,j}] = 0$ for all j and n, $E[X_{n,j}^2] = \frac{1}{n}$ for every j and n, the sum Y_n converges to a normal distribution but the limiting variance $\sigma^2 < 1$. Will Lindeberg's condition hold in this case?