Homework Set 3. Due March 1.

1. Consider a Markov process $x(t)$ on the set $\{1, 2\}$ where the transitions occur from $1 \to 2$ at rate λ and from $2 \to 1$ at rate μ. Write down differential equations for the transition probabilities $\{\pi_{i,j}(t) : i, j = 1, 2\}$. Solve them and calculate $\pi_{i,j}(t)$. Does the limit $\lim_{t\to\infty} \pi_{i,j}(t) = q_{i,j}$ exist ? What is it? Could you have guessed it?

2. A simple random walk is $S_n = X_1 + X_2 + \cdots + X_n$, $S_0 = 0$where $\{X_i\}$ are independent and equal ± 1 with probability $\frac{1}{2}$. The reflection principle says that for any ineger $k \geq 1$

$$P[\sup_{1\leq j\leq n} S_j \geq k] = 2P[S_n \geq k+1] + P[S_n = k]$$

Write down the equation to be satisfied by

$$u(n, k) = P[\sup_{1\leq j\leq n} S_j \geq k]$$

and check that $2P[S_n \geq k+1] + P[S_n = k]$ satisfies it.