Homework Set 8. Due April 12, 2004.

1. Let $\mathcal{L}$ be the Black and Scholes generator

$$\mathcal{L} = \frac{1}{2}\sum_{i,j} a_{i,j} x_i x_j D_{x_i} D_{x_j} + \sum_j \mu_j x_j D_{x_j}$$

If $u(t,x)$ is bounded on $R^d \times [0,T]$ by $C(1+\|x\|)^k$ for some k and is a smooth solution of

$$u_t = \mathcal{L}u$$

with $u(0,x) = 0$ for all x, then show that $u(t,x) \equiv 0$.

2. Show that the solution of the one dimensional SDE

$$dx(t) = x^2(t)dt + d\beta(t), x(0) = 0$$

becomes infinite at a finite (random) time τ. Can you get an upper bound on $E[\tau]$?