

Due Oct 9, 2000

Q1. Let $x(t)$ be a Gaussian process, with $E[X(t)] = 0$ and $\text{Cov}[x(t)x(s)] = \rho(s, t)$.

Let

$$\Delta(h) = \sup_{\substack{0 \leq s, t \leq T \\ |t-s| \leq h}} E[(x(t) - x(s))^2] = \sup_{\substack{0 \leq s, t \leq T \\ |t-s| \leq h}} [\rho(t, t) + \rho(s, s) - 2\rho(s, t)].$$

If

$$\Delta(h) \leq C[\log \frac{1}{h}]^{-\alpha}$$

for some C and $\alpha > 1$, show by the use of the GRR inequality that the Gaussian process can be constructed to have continuous paths.