Due Oct 9, 2000

Q1. Let $x(t)$ be a Gaussian process, with $E[X(t)] = 0$ and $\text{Cov}[x(t)x(s)] = \rho(s,t)$.

Let
\[
\Delta(h) = \sup_{0 \leq s, t \leq T \atop |t-s| \leq h} E[(x(t) - x(s))^2] = \sup_{0 \leq s, t \leq T \atop |t-s| \leq h} [\rho(t,t) + \rho(s,s) - 2\rho(s,t)].
\]

If
\[
\Delta(h) \leq C[\log \frac{1}{h}]^{-\alpha}
\]
for some $C$ and $\alpha > 1$, show by the use of the GRR inequality that the Gaussian process can be constructed to have continuous paths.