Real Variables Fall 2007.

Assignment 13. Due Dec 3

Problem 1. Prove Riesz's theorem for $C[0,1]$ directly along the following lines. Let

$$F(t) = \inf[\Lambda(f) : f \geq \mathbf{1}_{[0,t]}]$$

Show that $F(t)$ is nondecreasing in t. Show that $F(t)$ is right continuous, i.e. $F(t+0) = F(t)$. Let μ be the measure on $[0,1]$ such that

$$\mu([0,t]) = F(t)$$

[The existence of μ was a HW problem earlier. Assume it exists.] Show that $\Lambda(f) = \int f \, d\mu$ for all $f \in C[0,1]$.

Problem 2. How do you prove the uniqueness of the representing measure μ in Riesz's theorem in the general case of a nonnegative linear functional $\Lambda(f)$ on the space $C(X)$ of continuous functions on a compact metric space X?