Problem 1. If μ is a measure with $\mu(R) = 1$ on the Borel $\sigma-$field $\mathcal{B}(R)$, its *distribution function* is defined by

$$F(x) = \mu[(-\infty, x]] = \mu[\{y : -\infty < y \leq x\}]$$

Show that $F(x)$ is nondecreasing, continuous from the right and

$$\lim_{x \to -\infty} F(x) = 0, \lim_{x \to \infty} F(x) = 1.$$

Conversely show that given such a function F there is a unique μ such that

$$F(x) = \mu[(-\infty, x]]$$

Problem 2. Find a sequence $f_n(x)$ on $[0, 1]$ such that $f_n(\cdot) \to 0$ in measure with respect to the Lebesgue measure but $\limsup_n f_n(x) = 1$ for every x.

Problem 3.

Consider $f_n(x) = n^p x^n (1 - x)$ on $[0, 1]$. $\lim_{n\to\infty} f_n(x) = 0$ for $x \in [0, 1]$. Determine the values of p for which $\int_{[0,1]} f_n(x)dx \to 0$. Are these the same as those for which $\sup_n f_n(x)$ is integrable on $[0, 1]$?