Problem set 3.

1. Show that if

$$\sup_N \sum_{\substack{1\le i\le N\\1\le j\le N}} |a_{i,j}| < \infty$$

then

$$\sum_{1\le i<\infty} \Big(\sum_{1\le j<\infty} a_{i,j}\Big) = \sum_{1\le j<\infty} \Big(\sum_{1\le i<\infty} a_{i,j}\Big)$$

Give an example to show that the conclusion can fail with both sides being finite if

$$\sup_N \sum_{\substack{1\le i\le N\\1\le j\le N}} |a_{i,j}| = \infty$$

2. Show that in a metric space if a sequence $\{x_n\}$ does not converge to x then there is a subsequence of $\{x_n\}$ such that no further subsequence of this subsequence can converge to x. In other words if every subsequence of a sequence has a further subsequence that converges to x then the entire sequence converges to x.

3. Construct a complete metric d on the interval $(0, 1)$ such that $d(x_n, x) \to 0$ if and only if $|x_n - x| \to 0$,