Problemset 5.

1. A subset A is a G_δ subset if $A = \cap_{i=1}^{\infty} G_i$ is a countable intersection of open sets $\{G_i\}$. Show that every closed set is a G_δ set. If A is a G_δ subset of a complete metric space (X, d) show that there is metric D on A that induces the same convergence as d on A, but (A, D) is complete.

2. Given k pairwise disjoint closed sets $C_1, \ldots, C_k$ of X, show that there is a continuous function $f(x)$ on X with $f(x) = i$ on C_i.

3. If C is a closed subset of X and if $0 \leq f \leq 1$ is a continuous function on C can it always be extended as a continuous function on X satisfying $0 \leq f \leq 1$ on X?