Problemset 7.

Q1. Show that the measure μ that represents the bounded linear function $\Lambda(f)$ on the space $C(X)$ of bounded continuous functions on a compact space X, as $\Lambda(f) = \int_X f d\mu$ is unique. That is if μ_1, μ_2 are two countably additive measures defined on the Borel subsets of a compact metric space and if $\int_X f d\mu_1 = \int_X f d\mu_2$ for all bounded continuous functions on X, then $\mu_1 = \mu_2$ on the Borel σ-field. Assume that $X = [0,1]^d$, the d-dimensional cube.

Q2. $\Lambda(f)$ is a nonnegative bounded linear functional on $C[0,\infty)$. Assume $\Lambda(1) = 1$. Then Λ has a representation $\Lambda(f) = \int_{R^+} f d\mu$ if and only if Λ satisfies $f_n \downarrow 0 \Rightarrow \Lambda(f_n) \to 0$, i.e. the monotone convergence theorem holds. Why dont we need this condition in the compact case?

Q3. $\mathcal{X}$ is the Banach space of sequences $\xi = \{a_n\} : n \geq 1$, such that $\sum_{n=1}^{\infty} |a_n| p_n < \infty$ where p_n for $n \geq 1$ are some fixed positve numbers. $\|\xi\| = \sum_{n=1}^{\infty} |a_n| p_n$. What are the bounde linear functionals on $\mathcal{X}$? What is the dual norm?