Lower Bounds for Pinning Lines by Balls

Xavier Goaoc

Loria, France.

A line L is a transversal to a family F of convex objects in R^d if it intersects every member of F. If, in addition, L is an isolated point of the space of line transversals to F, we say that F is a pinning of L. Pinnings of lines naturally appear in various contexts: geometric transversal theory (Hadwiger- and Helly-type theorems), geometric optimization (locally-minimal enclosing cylinders), and robotics (grasping and fixturing).

A pinning F of a line L is minimal if no proper subset of F pins L. A minimal pinning of a line by disjoint balls in R^d has size at most $2d - 1$. This bound is known to be tight in two dimensions. We prove that the bound is tight in all dimensions.

The upper bound on the size of minimal pinnings yields Helly-type theorems for sets of line transversals to collections of disjoint balls whose geometric permutations can be suitably constrained. Our new lower bound implies a lower bound on the Helly number in these situations. In particular, in the case of disjoint congruent balls in R^d, we improve the lower bound from 5 to $2d - 1$. This narrows the gap with the known upper bound of $4d - 1$ and shows that the Helly number is linear in d. This is the first lower bound for this problem that increases with the dimension, which incidentally answers a question of Danzer from 1957.

Joint work with Otfried Cheong and Andreas Holmsen.

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1