Geometry Seminar
Tuesday, December 2, 2008
Room 201 WWH at 6:00 P.M.

Random matrices:Universality of the spectral distribution and the circular law

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Given a $n \times n$ complex matrix A, let $\mu_A(x, y)$ be the counting measure generated by the (complex) eigenvalues of A.

We consider the limiting distribution (both in probability and in the almost sure convergence sense) of the normalized ESD $\mu_{\frac{1}{\sqrt{n}}A_n}$ of a random matrix $A_n = (a_{ij})_{1 \le i,j \le n}$ where the random variables $a_{ij} - E(a_{ij})$ are iid copies of a fixed random variable x with unit variance. We prove a **universality principle** for such ensembles, namely that the limit distribution in question is **independent** of the actual choice of the atom variable x. In particular, in order to compute this distribution, one can assume that x is real or complex gaussian. As a related result, we show how laws for this ESD follow from laws for the singular value distribution of $\frac{1}{\sqrt{n}}A_n - zI$ for complex z.

As a corollary we establish the Circular Law conjecture (in both strong and weak forms), that asserts that $\mu_{\frac{1}{\sqrt{n}}A_n}$ converges to the uniform measure on the unit disk when the a_{ij} have zero mean. (In particular, this strengthens the result I discussed in a colloquium at NYU in November 2007)

The proof uses tools from additive combinatorics, probability and high dimensional geometry.

(The talk is based on a recent paper "Random matrices: Universality of ESDs and the circular law", by T. Tao and V. Vu, with an appendix by M. Krisnapur.)

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.