

Geometry Seminar
Tuesday, October 6, 2009
Room 202 WWH at 6:00 P.M.

Visualizing \mathbb{R}^N and some new dualities.

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With parallel coordinates the perceptual barrier imposed by our 3-dimensional habitation is breached enabling the visualization of multidimensional objects. The highlights, interlaced with interactive demonstrations, are intuitively developed. M -dimensional objects are recognized recursively from their $(M - 1)$ -dimensional subsets. It emerges that a hyperplane in N -dimensions is represented by $(N - 1)$ indexed points. Points representing lines have two indices, those representing planes three indices and so on. In turn, this yields powerful geometrical algorithms (e.g. for intersections, containment, proximities) and applications including classification.

A smooth surface in 3-D is the envelope of its tangent planes each represented by 2 planar points. As a result it is represented by two planar regions, and a hypersurface in N -dimensions by $(N - 1)$ regions. This is equivalent to representing a surface by its normal vectors. Developable surfaces are represented by curves revealing the surface characteristics. Convex surfaces in any dimension are recognized by hyperbola-like regions. Non-orientable surfaces yield stunning patterns unlocking new geometrical insights.

Non-convexities like folds, bumps, concavities are visible. The patterns persist in the presence of errors. Intuition gained from the \mathbb{R}^3 representations leads to generalization for \mathbb{R}^N with beautiful new dualities like **cusp** in $\mathbb{R}^N \leftrightarrow (N - 1)$ **swirls** in \mathbb{R}^2 , **twist** in $\mathbb{R}^N \leftrightarrow (N - 1)$ **cusps** in \mathbb{R}^2 .

For more information please visit the seminar website at:
http://www.math.nyu.edu/seminars/geometry_seminar.html.