

Geometry Seminar  
Tuesday, Nov 08, 2011  
Room 512 WWH at 6:00 P.M.

# Existence of small families of $t$ -wise independent permutations and $t$ -designs via local limit theorems

Shachar Lovett  
IAS, Princeton.

We show existence of rigid combinatorial objects that previously were not known to exist. Specifically, we consider two families of objects:

1. A family of permutations on  $n$  elements is  $t$ -wise independent if it acts uniformly on tuples of  $t$  elements. Constructions of small families of  $t$ -wise independent permutations are known only for  $t=1,2,3$ . We show that there exist small families of  $t$ -wise independent permutations for all  $t$ , whose size is  $n^{O(t)}$ .
2. A  $t$ - $(v,k,\lambda)$  design is a family of sets of size  $k$  in a universe of size  $v$  such that each  $t$  elements belong to exactly  $\lambda$  sets. Constructions of  $t$ -designs are known only for some specific settings of parameters. We show that there exist small  $t$ -designs for any  $t,v,k$  whose size (and  $\lambda$ ) are  $v^{O(t)}$ .

Both results follow the same methodology. We formulate the problem as a random walk on a lattice with a prescribed set of allowed steps, and then study the random walk using local limit theorems.

Joint work with Greg Kuperberg and Ron Peled

For more information please visit the seminar website at:  
[http://www.math.nyu.edu/seminars/geometry\\_seminar.html](http://www.math.nyu.edu/seminars/geometry_seminar.html).