Geometry Seminar Tuesday, September 20, 2011 Room 512 WWH at 6:00 P.M.

Polyhedral geometries and graphs for crystallographic groups

Egon Schulte, Northeastern University, Boston

The study of highly symmetric discrete structures in ordinary euclidean 3-space has a long and fascinating history tracing back to the early days of geometry. With the passage of time, various notions of polyhedral structures have attracted attention and have brought to light new exciting figures intimately related to finite or infinite groups of isometries.

A radically new, skeletal approach to polyhedra in space was pioneered by Branko Grunbaum in the 1970's, building on Coxeter's work. A polyhedron is viewed as a finite or infinite periodic geometric (edge) graph in space equipped with additional structure imposed by the faces, and its symmetry is measured by transitivity properties of its geometric symmetry group. Since the mid 1970's, there has been a lot of activity in this area, beginning with the full enumeration of the "new" regular polyhedra by Branko Grunbaum and Andreas Dress around 1980, moving to the full enumeration of chiral polyhedra around 2005, and continuing with the enumeration of certain classes of regular polyhedra and polytopes in higher-dimensional spaces by Peter McMullen.

While all these structures have the essential characteristics of polyhedra and polytopes, the more general class of discrete "polygonal complexes" in space is a hybrid of polytopes and incidence geometries. In very recent joint work with Daniel Pellicer, a complete classification of the regular polygonal complexes was obtained. These are periodic structures with crystallographic symmetry groups exhibiting interesting geometric, combinatorial, and algebraic properties.

We survey the present state of the ongoing program to classify discrete polyhedral structures in space by distinguished transitivity properties of their symmetry groups.

For more information please visit the seminar website at: http://www.math.nyu.edu/seminars/geometry_seminar.html.