Geometry Seminar
Tuesday, Feb 3, 2009
Room 317 WWH at 6:00 P.M.

Small-size Epsilon-Nets for Geometric Range Spaces

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Since their introduction in 1987 by Haussler and Welzl, ϵ-nets have become one of the central concepts in computational and combinatorial geometry, and have been used in a variety of applications, such as range searching, geometric partitions, and bounds on curve-point incidences. In particular, they are strongly related to geometric set-cover and hitting-set problems.

A range space (or a hypergraph) (X,R) is a pair consisting of an underlying universe X of objects, and a certain collection R of subsets of X (also called ranges). Given a range space (X,R), and a parameter $0 < \epsilon < 1$, an ϵ-net for (X,R) is a subset N of X with the property that any range that captures at least ϵ-fraction of X contains an element of N. In other words, N is a hitting set for all the "heavy" ranges.

Of particular interest are geometric range spaces, since then they admit small-size ϵ-nets. Specifically, the Epsilon-Net Theorem of Haussler and Welzl asserts that in this case there exists an ϵ-net of size $O(1/\epsilon \log 1/\epsilon)$. One of the major questions in the theory of ϵ-nets, open since their introduction more than 20 years ago, is whether the factor $\log 1/\epsilon$ in the upper bound on their size is really necessary, especially in typical low-dimensional geometric situations. A central motivation then arises from the technique of Bronnimann and Goodrich to obtain, in polynomial time, improved approximation factors for the geometric hitting-set and set-cover problems: The existence of an ϵ-net of size $O((1/\epsilon)f(1/\epsilon))$, for some slowly-growing function $f(\ldots)$, implies an approximation factor of $O(f(OPT))$, where OPT is the size of the smallest such set.

In this talk I will survey some of the fundamental results concerning small-size ϵ-nets. I will then discuss range spaces of points and axis-parallel boxes in two and three dimensions, and show that they admit an ϵ-net of size $O(1/\epsilon \log\log 1/\epsilon)$, and also present several extensions to "fat" planar ranges.

Joint work with Boris Aronov (Polytechnic Institute of NYU) and Micha Sharir (Tel Aviv University).

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http://www.math.nyu.edu/seminars/geometry_seminar.html.