

Geometry Seminar  
Tuesday, January 27, 2009  
Room 317 WWH at 6:00 P.M.

# Hidden Structures in the Family of Convex Functions in $R^n$ and the New Duality Transform.

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We discuss in the talk an unexpected observation that very minimal basic properties essentially uniquely define some classical transforms which traditionally are defined in a concrete and quite involved form.

We start with a characterization of a very basic concept in Convexity and Functional Analysis: Duality and the Legendre transform. We show that the Legendre transform is, up to linear terms, the only involution on the class of convex lower semi-continuous functions in  $R^n$  which reverses the (partial) order of functions. This leads to a different understanding of the concept of duality, and which we then apply also to many other well known settings. It is also true that any involutive transform (on this class) which exchanges summation with inf-convolution, is, up to linear terms, the Legendre transform.

At the same time, considering the class of non-negative convex functions (with 0 value at 0), changed the picture and brings an additional, new duality for this class, which was not considered before. We will study this new duality and discuss its properties. In particular, this leads to new structures on this class.

The classical Fourier transform may be also defined (essentially) uniquely by the condition of exchanging convolution with product together with the form of the square of the transform (the last fact is a joint work also with Semyon Alesker).

For more information please visit the seminar website at:  
[http://www.math.nyu.edu/seminars/geometry\\_seminar.html](http://www.math.nyu.edu/seminars/geometry_seminar.html).