Geometry Seminar
Tuesday, Feb 17, 2009
Room 317 WWH at 6:00 P.M.

A Taste of Algebraic Combinatorics for (Combinatorial) Geometry

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An arrangement of hyperplanes, A, in real space dissects the space into "regions". In complex space it creates non-trivial cohomology. An arrangement has a natural "dual" arrangement. The number of (real) regions and the number of bounded regions, and the (complex) Betti numbers, and the similar numbers for the dual, all satisfy four general properties called "Deletion-Contraction", "Multiplication", "Invariance", and "Unitarity". The most important of these, Deletion-Contraction, is the property of a function F that

$$F(A) = F(A \backslash e) + F(A/e),$$

where e is (almost) any one of the hyperplanes, $A \backslash e$ is A without e, and A/e is the arrangement induced in the subspace e. There is a complete theory of the functions of a hyperplane arrangement that satisfy these properties. The theory generalizes to, and is best understood through, matroid theory.

Recently there has been much interest, due to knot theory and physics, in a more general property, "Parametrized Deletion-Contraction". Here one assigns two fixed "parameters", d_e and c_e, to each hyperplane e and requires that

$$F(A) = d_e F(A \backslash e) + c_e F(A/e).$$

Joanna Ellis-Monaghan and I have been developing the general theory of functions that satisfy Parametrized Deletion-Contraction (and not necessarily anything else) through commutative algebra. I will survey our results.

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.