

Geometry Seminar  
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Room 201 WWH at 6:00 P.M.

# A complex analogue of Toda's theorem.

Saugata Basu  
Purdue University

Toda proved in 1989 that the (discrete) polynomial time hierarchy,  $\mathbf{PH}$ , is contained in the class  $\mathbf{P}^{\#\mathbf{P}}$ , namely the class of languages that can be decided by a Turing machine in polynomial time given access to an oracle with the power to compute a function in the counting complexity class  $\#\mathbf{P}$ . This result which illustrates the power of counting is considered to be a seminal result in computational complexity theory. An analogous result (with a compactness hypothesis) in the complexity theory over the reals (in the sense of Blum-Shub-Smale machines) was obtained by the author and Thierry Zell in 2008. Unlike Toda's proof in the discrete case, which relied on sophisticated combinatorial arguments, the proof of this result is topological in nature in which the properties of the topological join are used in a fundamental way. However, the constructions used are semi-algebraic in nature – they use real inequalities in an essential way, and as such do not extend to the complex case. In this talk, I will explain how to extend the techniques developed in the prior paper to the complex case. A key role is played by the complex join of quasi-projective complex varieties. As a consequence we obtain a complex analogue of Toda's theorem.

For more information please visit the seminar website at:  
[http://www.math.nyu.edu/seminars/geometry\\_seminar.html](http://www.math.nyu.edu/seminars/geometry_seminar.html).