Geometry Seminar
Tuesday, Feb 08, 2010
Room 201 WWH at 6:00 P.M.

Inequalities Between the Number of Points, Lines, Planes, *etc.* Spanned by n Points.

George Purdy,
University of Cincinnati

Given n points, let L, p, and h be the number lines, planes, and hyperplanes spanned by the n points.

Assuming that certain degenerate configurations are avoided, we know that

1. $L > n$ in $\mathbb{R}^2$,
2. $p > n$ in $\mathbb{R}^3$, and
3. $h > n$ in $\mathbb{R}^d$.

(1) follows from Sylvester's Theorem, and (2) and (3) were proved by Motzkin in 1951.

But in fact, as we now know, all of these are true in the context of simple matroids. In particular, they are true for $d > 2$ in the space F^d for any finite field $F = GF(p^k)$.

Some while ago I was playing around with a conjecture of mine involving n, L, and p, and Erdós asked me "why don't you just ask when $p > L$?"

Using the Szemeredi-Trotter result, I was able to show that, in $\mathbb{R}^3$

(P_3) $p > cL$, provided the points do not all lie on two skew lines, or on a plane.

Unlike (1), (2), and (3), (P_3) is false in F^3, for sufficiently large finite fields F. This may be seen by taking all of the points of the space as the n points, and computing the Gaussian coefficients that give n, L, and p.

So you might say that P_3 captures a property of $\mathbb{R}^3$, but this isn't quite true, because I can also prove P_3 in complex $\mathbb{C}^3$.

I naturally conjecture that in $\mathbb{R}^4$:

(P_4) $h > cp$ if certain configurations are avoided, with an appropriate generalization to $\mathbb{R}^d$.

Two students Justin Smith and Ben Lund and I are gradually proving P_d. Our first task was to develop some tools, and the Szemeredi-Trotter theorem in higher dimensions is not the right tool, as it turns out. I hope to be able to outline a proof of (P_d) by the time of the talk.

For more information please visit the seminar website at:
`http://www.math.nyu.edu/seminars/geometry_seminar.html`.