Affine Quermassintegrals and Minkowski Valuations

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The Blaschke-Santaló and the polar Petty projection inequality are two of the best known and most powerful affine isoperimetric inequalities in convex geometric analysis. In particular, they are significantly stronger than the classical Euclidean Urysohn and *the* isoperimetric inequality, respectively. In 1988, Lutwak conjectured that for convex bodies in $\mathbb{R}^n$ and each $1 \leq i \leq n-1$ an affine isoperimetric inequality holds for the socalled ith affine quermassintegral. The special cases $i = 1$ and $i = n-1$ being the Blaschke-Santaló and the polar Petty projection inequality, respectively.

In this talk, we present new isoperimetric inequalities for Minkowski valuations intertwining rigid motions of degree $i = 1$ and $i = n-1$. These inequalities not only improve the Urysohn and *the* isoperimetric inequality but interpolate between these classical Euclidean inequalities and the Blaschke-Santaló and the polar Petty projection inequality, respectively. Moreover, among these large families of isoperimetric inequalities, the affine ones turn out to be the strongest inequalities. Finally, we also relate the polar volume of Minkowski valuations of degree $2 \leq i \leq n-2$ to Lutwak's conjecture on affine quermassintegrals.