OCCUPATION MEASURES, COMPACTNESS AND LARGE DEVIATIONS

In a reasonable topological space, large deviation estimates essentially deal with probabilities of events that are asymptotically (exponentially) small, and in a certain sense, quantify the rate of these decaying probabilities. In such estimates, lower bound for open sets and upper bound for compact sets are essentially local estimates. However, upper bounds for all *closed sets* often require compactness of the ambient space or stringent technical assumptions (e.g., exponential tightness), which is often absent in many interesting problems which are motivated by questions arising in statistical mechanics (for example, distributions of occupation measures of Brownian motion in the full space $\mathbb{R}^d$).

Motivated by problems that carry certain shift-invariant structure, we present a robust theory of "translation-invariant compactification" of orbits of probability measures in $\mathbb{R}^d$. This enables us to prove a desired large deviation estimates on this "compactified" space. Thanks to the inherent shift-invariance of the underlying problem, we are able to apply this abstract theory painlessly and solve a long standing problem in statistical mechanics, the mean-field polaron problem.

This is based on joint works with S. R. S. Varadhan, as well as with Erwin Bolthausen and Wolfgang König.