

Hypoelliptic Laplacian and probability
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The hypoelliptic Laplacian is a family of operators, indexed by $b \in \mathbf{R}_+^*$, acting on the total space of the tangent bundle of a Riemannian manifold, that interpolates between the ordinary Laplacian as $b \rightarrow 0$ and the generator of the geodesic flow as $b \rightarrow +\infty$. The probabilistic counterpart to the hypoelliptic Laplacian is a 1-parameter family of differential equations, known as geometric Langevin equations, that interpolates between Brownian motion and the geodesic flow.

I will present some of the probabilistic ideas that explain some of its remarkable and often hidden properties. This will include the Itô formula for the corresponding hypoelliptic diffusion, and the corresponding Malliavin calculus.

I will also explain some of the applications of the hypoelliptic Laplacian that have been obtained so far.