Block Spin Transformations of 2D Sigma Model,
Toward Solving a Millennium Problem

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Abstract: We analyze 2D O(N) sigma model by block spin transformation:

$$\exp\left[-\frac{1}{2}\langle\phi, G_0^{-1}\phi\rangle - \frac{g_0}{2N}\sum_x(\phi^2(x)-N\beta)^2\right]$$

where $\phi(x)=(\phi_1(x),\cdots,\phi_N(x))$, $G_0^{-1}=-\Delta+m_0^2$ and $-\Delta$ is the lattice laplacian on Z^2. Put $\phi_0=\phi$ and recursively define block spins

$$\phi_{n+1}(x)=\frac{1}{L^2}\sum_{|\zeta|<L/2}\phi_n(Lx+\zeta)$$

We represnt ϕ_n in terms of block-spin ϕ_{n+1} of next order and fluctuations z_n. z_n is a massive Gaussian but gets strong effects of back ground field ϕ_{n+1}. We define a new notion of domain walls in the sigma model which has O(N) symmetry. The domain wall regions D_w have high energies, and on $(D_w)^c$ we can safely implement the block spin transformations. Thus we obtain the renormalization group flow of the 2D sigma model, which enables us to prove our long-standing claim.