Large-N analysis of the sinh-model

K. K. Kozlowski, IMB, UMR 5584 du CNRS, France.

The quantum separation of variables is a tool allowing one to study numerous quantum mechanical models of N-particles interacting in one dimension. The simplest yet non-trivial example corresponds to the quantum Toda chain. It turns out that for models solvable by the quantum separation of variables, numerous observables of interest to their physics can be expressed in terms of N-fold multiple integrals which take the general form

$$\int_{C^N} \prod_{a<b}^{N} \Big\{ \sinh\big[\frac{\pi}{\omega_1}(\lambda_a - \lambda_b)\big] \cdot \sinh\big[\frac{\pi}{\omega_2}(\lambda_a - \lambda_b)\big] \Big\} \cdot \prod_{a=1}^{N} \Big\{ e^{-V(\lambda_a)} \Big\} \cdot d^N\lambda \ .$$

The choice of the curve C, of the periods ω_1, ω_2 and of the confining potential V depends on the model. The analysis of the large-N asymptotic behaviour of these integrals is primordial for understanding the limit to quantum models of infinitely many interacting particle, and can provide a better understanding of what a quantum field theory actually is from the mathematical point of view. In this talk, I shall report on recent developments in the large-N analysis of such integrals when $C = \mathbb{R}$. In particular, I will present the form taken by the asymptotics for the specific example $V(\lambda) = \sigma \cosh(\lambda)$. The material I will present in this talk stems, in part, from a joint work with G. Borot and A. Guionnet.