Exercise 1. Let $X$ be a nonnegative random variable with null expectation. Prove that it is 0 almost surely.

Exercise 2. Let $X$ be a random variable in $L^1(\Omega, \mathcal{A}, \mathbb{P})$. Let $(A_n)_{n \geq 0}$ be a sequence of events in $\mathcal{A}$ such that $\mathbb{P}(A_N) \rightarrow 0$. Prove that $\mathbb{E}(X1_{A_n}) \rightarrow 0$.

Exercise 3. Let $X$ be a Gaussian random variable with expectation 0 and variance $\sigma^2$. What is $\mathbb{E}(X^3)$? What is $\mathbb{E}(X^4)$?

Exercise 4. Let $(X,Y)$ be chosen uniformly on the triangle $\{(x,y) \in \mathbb{R}^2 : x+y \leq 1, x \geq 0, y \geq 0\}$. What is the density function of $(X,Y)$? Find the distributions of $X+Y$, $X-Y$, $XY$.

Exercise 5. A samouraï wants to create a triangle with a (rigid) spaghetti. With his saber, he cuts this spaghetti on two places, chosen uniformly and independently along this traditional pasta. What is the probability that he can create a triangle with sides these three pieces of spaghetti?

Exercise 6. Assume that $X_1, X_2, \ldots$ are independent random variables uniformly distributed on $[0,1]$. Let $Y^{(n)} = n \inf\{X_i, 1 \leq i \leq n\}$. Prove that it converges weakly to an exponential random variable, i.e. for any continuous bounded function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$,
\[
\mathbb{E} \left( f(Y^{(n)}) \right) \rightarrow_{n \rightarrow \infty} \int_{\mathbb{R}^+} f(u)e^{-u}du.
\]