Teaching Statement

BY Joseph Coffey

I enjoy teaching. Mathematical research is a blessing, both beautiful and rewarding, but it can be a lonely pursuit. I find teaching a welcome antidote. I have benefitted from the time and effort of a great many teachers, and I owe my students that same time and effort.

I began teaching as an undergraduate when I served as a TA in two physics courses at Berkeley High. Then as a graduate student at Stony Brook I served as TA in a diverse range of classes, and at Courant I taught a series of three courses: Pre-Calculus, Business Calculus, and Calculus I (a standard calculus for scientists course). However, teaching is not limited to formal lectures. It is a practice, part of the daily flow of academia. We learn from our mentors and colleagues, and give back to those around us. At Courant I have had the pleasure of the Symplectic seminar. I have tried to serve there and elsewhere as a bridge between the graduate students and the tenured faculty. I look forward to continuing this mentoring at my next position. I have also enjoyed being the representative topologist among these symplectic analysts, giving a series of lectures on convex surfaces in contact manifolds, and other topological constructs within the field.

Most of my undergraduate teaching has been within the calculus sequence. Calculus has a complex role in the undergraduate curriculum. It serves both as technical course for those whose disciplines require it, and as part of the broad training of a liberal education. Combining these two directives challenges the instructor to provide a course that aids them both in proficiency of technique, and in forming a flexible mind. There is a tension in learning between the two. Repetitive exercises builds comfort and technique, giving the students solid building blocks. Repetetion is especially important because most people, unlike mathematicians, think inductively rather than deductively. So without some repetition of application a statement has no meaning for them. Thus, in order to develop technique, most high school are trained to follow recipes. Indeed, when they arrive to university they often view mathematics as a collection of these prescriptions. As a result they often feel that math is a stale subject, but at the same time departure from these formulaic exercises frighten them. They have no process, no set of habits, to aid them outside these narrow confines. Our job is to provide these and free them from this trap between boredom and fear.

This isn't easy, and in truth it is necessary to have realistic goals. Simple exposure to more difficult problems can breed fear, for the students have been trained to look at a problem and try to recall the appropriate recipe. If they cannot remember one quickly they give up. If there are too many problems they haven't been given precise formulas for solving they become angry with themselves and the instructor. This recipe recalling is a natural consequence of the training they received in high school, but it leads to brittle thinking.

One of my goals in the classroom is to replace this process of recall with a new more flexible one. In [Pol48] Polya described a list of questions that most mathematicians ask themselves when confronted with a difficult problem:

1. UNDERSTAND THE PROBLEM You have to understand the problem. * What is the unknown? What are the data? What is the condition? * Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory? * Draw a figure. Introduce suitable notation. * Separate the various parts of the condition. Can you write them down?

2. DEVISING A PLAN * Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution. * Have you seen it before? Or have you seen the same problem in a slightly different form? * Do you know a related problem? Do you know a theorem that could be useful? * Look at the unknown! And try to think of a familiar problem having the same or a similar unknown. * Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible? .... (The list continues from there)

It has the advantage of both being concrete and thus comforting, but also general and thus flexible, with wide applications even outside mathematics. I have only begun trying to work this into my classes, but I feel there are many opportunities. For example one can:

1. Use it repeatedly in lecture on examples. Show how the recipes they may recall are specializations of this process.
2. Guide students through it in section.
3. Give partial credit on exams/home work for applying such a process to a question.

Obviously this isn't a panacea; there are none of these in teaching, but the goal is to brings the students in more direct conversation with math, to help them face more difficult problems with the comfort of a method to go about it.

One the reason students struggle is that they do not have enough respect for their own understanding or difficulties. Having given up on understanding, they try to learn to pretend to understand. This is nearly impossible. Thus we must help the students learn to explain what it is they understand and what confuses them. During lecture I often ask random students to re-explain what I did, in a variation of Gelfand's practice in his seminar. This has to be done with smile, but its goal is both to ensure that we are communicating and to teach the student the habit of checking his own understanding. I also usual ask my students one essay question per exam. The question is given to the students ahead of time, and centers on the most important idea of that section of the course, such as the Fundamental Theorem of Calculus. Without direct effort, very few students will internalize these ideas.

Within the classroom my main role is to humanize and embody mathematics, to be someone that the students can emulate. To do this you must allow yourself to be human. The more difficult the mathematics the more important this dictum becomes. Thus, I have done epsilons and deltas in a funny hat. I believe that class should be fun, but even more important than humor is that the students sense your passion, and that there is mathematics beyond calculus. I try at some point in the semester to talk for a few minutes about what I do. Even though this is possible in only its barest outlines, the students usually have some personal curiosity of how I spend my day. I want them to sense some trail from calculus out into the mathematical wilderness. A trail they might choose to take, or merely admire.

Unfortunately there will always be some students who fail calculus. The background of the students is too diverse, and the pace of a college class is too quick to avoid this, but I feel it is vital for the instructor to make this failure as positive experience as is possible. It is vital to show them that while unfortunate this is not the end of their career, and that I still believe in their eventual success. I keep 3-4 copies of *How to Ace Calculus* [Ada98] – a textbook companion aimed at interpreting calculus in a friendly framework – in my office for these troubled students. After the first midterm I loan them these books. Occasionally this (and other) attention is sufficient to help them recover. But regardless I think it keeps them from quitting, and from feeling worthless about the experience. It helps them use this semesters failure to build a foundation for success next semester.

Finally I would like to discuss a technical approach to teaching. While a teacher serves an invaluable role as model and coach, it is impossible for one to provide the continuous support to each student that is sometimes required. Amit Sahai, a theoretical computer scientist at UCLA, and I have worked off and on a project to develop a computer tutoring system to provide this support. The main difficulty in learning mathematics, is that one has to be asked the proper question, or told the proper idea, *at the proper time*. This time and question differs for each student. Our system aims to discover what the student knows, and what would be the best next question or idea.

More precisely we model Mathematics as a directed graph. A calculus question such as "What is the derivative of $\sin(x^2)$?" sits as a node in this graph. If a student cannot answer this question it is probable that he cannot answer one of the following "parent" questions: What is the derivative of $\sin(x)$? What is the derivative of x^2?, or that he doesn't know the chain rule. On the other the question is a parent of more complicated computations of derivatives, and perhaps other questions. In this way this question sits an a directed graph of countless other questions and concepts within mathematics. Then for each student one decorates these nodes with some expected probabilities that he can answer it. If he fails a question, we can damp the probabilities of his parents, if he succeeds we can raise them. The idea is to make use of this network of probabilities to endow the computer with a picture of what the student knows. Now in the naive implementation we carried out 5 years ago the system had some promising and some irritating properties. Such a network of mathematical connections, while important to the learning process, is not the whole picture. But it may not be infinitely far off. It is possible that with sufficient work we might make a meaningful contribution. Perhaps we can find a grant to serve as an umbrella for this sort of work, and get a couple of (probably computer science) graduate students excited.

Bibliography

[Ada98] C. Adams. *How to ace calculus*. W.H. Freeman and Company, 1998.

[Pol48] G. Polya. *How to Solve It. A New Aspect of Mathematical Method*. Princeton University Press, Princeton, N. J., 1948.