

24th, both preceded by heavy swell and followed by low dew-point. The north wind evidently precipitated the terrific downpour on north Hilo. There was lightning reported from Hawaii for the 8th, 9th, 13th, 24th, and Maui on the 8th. Snow fell on Mauna Kea and Mauna Loa on the 8th and 24th; on Haleakala on the 8th or 9th. Earthquake reported at Hilo, 7:30 p. m. on the 2d.

AN AURORAL-LUNAR HALO DISPLAY.

H. H. TEN BROECK, Braidentown, Fla., dated December 29, 1901.

At midnight of the 28th I observed an auroral display with an axis extending about west-northwest to east-southeast. There was at the time an unusually brilliant halo around the moon, about 45° in diameter. The upper half was fringed on the edge by rays about 3° or 4° long, a few much longer, radiating, not from the moon (the center of the halo), but from a point below it about east-southeast on the horizon. Some of the rays extended northward as far as Cassiopeia while the Pleiades were a little south of the center of the long bands, which converged toward a point about west-northwest on the horizon, although not reaching it by 10° or 15°. The bands and the rays on the halo appeared and disappeared slowly like auroral bands and were of a pale white color. There was an 8 or 10 mile southeasterly wind blowing and half of the sky was covered with very light clouds of a cirro-cumulus order, with an almost imperceptible motion. Very few of the bands extended below the moon. The fringe of rays on the upper edge of the halo was well marked and closely resembled that often seen on an auroral arch; it, too, changed in brightness, slowly, as well as in length. It was a well-marked auroral display, with the moon's halo as a starting point. In half an hour the halo and bands had disappeared as well as most of the clouds.

I have never before seen or heard of an aurora from a lunar halo, nor one with its middle line running west-northwest and east-southeast.

MEXICAN OLIMATOLOGICAL DATA.

Through the kind cooperation of Señor Manuel E. Pastrana, Director of the Central Meteorologic-Magnetic Observatory, the monthly summaries of Mexican data are now communicated in manuscript, in advance of their publication in the Boletin Mensual. An abstract, translated into English measures, is here given, in continuation of the similar tables published in the MONTHLY WEATHER REVIEW since 1896. The barometric means are now reduced to standard gravity.

Mexican data for December, 1901.

Stations.	Altitude.		Temperature.			Relative humidity.	Precipitation.	Prevailing direction.	
	Feet.	Inch.	Max.	Min.	Mean.			Wind.	Cloud.
Chihuahua	4,069	25.33	80.6	23.0	53.8	47	ne.
Guadalajara	5,186	24.90	83.7	40.1	58.1	51	n.
(Obs. del Est.)									
Guanajuato	6,640	23.66	79.2	30.7	57.0	49	ws.
Leon (Guanajuato)...	5,906	24.27	73.6	28.6	54.7	59	0.05	nw.
Mazatlan	25	29.89	79.5	55.9	70.0	71	T.	nw.	w.
Merida	50	29.94	83.2	48.9	71.4	75	0.16	ne.
Mexico (Obs. Cent.)...	7,472	23.01	71.2	30.9	52.5	51	T.	n.
Monterrey (Sem.)...	1,626	28.20	97.7	31.1	57.9	54	ne.
Morelia (Seminario)...	6,401	23.91	73.0	30.2	54.9	60	sw.
Puebla (Col. Cat.)...	7,125	23.30	71.6	35.6	55.6	57	0.03	e.
Puebla (Col. d. Est.)...	7,118	23.32	72.3	36.6	53.6	60	0.06	ene.
Saltillo (Col. S. Juan)	5,399	24.76	72.7	28.9	51.4	55	sw.
Toluca	8,812	21.92	68.7	20.1	47.1	55	0.05	w.
Tuxtla (Gutierrez, Chiapas)...	1,864	28.10	94.6	47.8	71.6	72	nw.
Zapotlan	5,078	25.05	77.0	41.7	60.4	51	T.	n.

* Reduced to standard temperature and gravity.

THE PHYSICAL BASIS OF LONG-RANGE WEATHER FORECASTS.¹

By PROF. CLEVELAND ABBE.

The expression "long range" must not be misunderstood. It refers only to the length of time intervening between the date of making a weather prediction and the date when we expect it to be fulfilled. At the present time, by the help of the daily weather map, the official weather forecasters of this country, and indeed of every civilized nation on the globe, publish forecasts, in detail, of approaching weather changes, and especially storms, for one and two, or possibly occasionally three days in advance. These predictions all relate to comparatively minute details for regions that have been charted and studied daily for many years. They merely represent the direct teachings of experience; they are generalizations based upon observations but into which physical theories have as yet entered in only a superficial manner if at all. They are, therefore, quite elementary in character as compared with the predictions published by astronomers, based on the laws of gravitation and inertia, or the predictions sometimes offered by chemists, based on the laws that are being worked out by these investigators. Even the electrician, familiar with mathematical physics ventures on predictions based on far more complex theories than are as yet at the command of the meteorologists. But the latter are slowly building a grand structure, mathematical, graphical, and numerical, in which deductive reasoning will take the place of empirical rules. The whole will eventually form a complex intellectual machine, by means of which the general, and possibly the detailed phenomena of the atmosphere, will be followed up day by day. Then we shall be justified in calling our work rational science, as distinguished from empirical science. I use the word science in its fundamental meaning, as referring to that of which we have accurate knowledge, and not that which is purely speculative.

While I thus indulge hope in the prospective future high perfection of the science of the weather, I recognize the fact that we must not expect to realize these hopes in this generation. The progress of all science is necessarily slow. From Copernicus to Kepler, from Kepler to Newton, from Newton to La Place, and from La Place to the living giants in the theoretical astronomy of to-day, we proceed by steps of a century each. In chemistry, from Berzelius to the present day, we have scarcely one such step. In electrical science we are less than a century distant from Ohm and Green. In meteorology, considered as an application of physics, we begin with Espy's work of 1830, but considered as a branch of mathematical science we begin with Ferrel's work of 1856. The development of a correct "Theoria meteorologica" has made good progress during the past twenty years, but we are still at work on the introductory chapter. Some would hasten the work by unnatural stimulants in response to the feverish anxiety of the people and the daily newspapers, but we must be content to await the surer results of a slow but natural growth. Personally, I hope I may live to see the day when some of our universities will offer attractive courses in dynamic, experimental, and observational meteorology to advanced students of mathematics and physics, when those who are prepared to profit by such lectures may in their turn contribute to the advancement of our knowledge. It will not do for us to be so absorbed in so-called practical work as to neglect the research work that is still more practical. The practical work of to-day is but the application of the results of the past research. The research of to-day will be the basis

¹This paper is a summary of lectures delivered at Johns Hopkins University in February, 1901. It was prepared for the meeting of the American Association for the Advancement of Science, Denver, August, 1901, and is now first published.

of the practical work of the future, and so on, step by step. At the present time this important branch of physical and mathematical science is recognized in German and French universities, but has not yet taken its proper place in American or English institutions. It demands not merely a few lectures or an occasional mention, but the complete devotion of at least one professor and one post graduate student annually.

At the present time it is, of course, impossible to make long-range forecasts of the details that make up the daily weather, but there is a large class of the community that would be greatly benefitted and perhaps perfectly satisfied if we could forecast the general features of a season, such as a week, or a month, a spring or summer, an autumn or winter. This class of work we call seasonal or climatological forecasts. It is desired to know whether on the average there will be many frosts or severe freezing weather, whether the range of temperature will be large or small, whether it will be especially cloudy or sunny, whether the rain or snow will be abnormally large or small, or whether having fallen in abundance it will evaporate too rapidly to be utilized for irrigation. Such seasonal forecasts have been considered as a possibility, even though accurate long-range weather predictions may not be so. The anxiety of farmers with regard to their future crops has led them from time immemorial to look about for natural signs, and they find them everywhere. If the moon is in a certain position, they say that it indicates a wet month; if the wild geese fly very high on their journey northward, it indicates a dry summer; if the woodchuck goes back into his burrow, it means a long, cold spring; if the celestial milky way happens to trend in a northwest direction when the farmer happens to look at it, it means a prevalence of cold, northwest winds; if the trees leaf out early, it means an early spring; if the fur of the wild animals is unusually thick, or if a squirrel is laying up a large store of food, it means a long, hard winter; and so we might go on indefinitely, as though the animals and even the trees know more about the laws of nature than man himself. When man stoops to learn from these about matters that he ought to study out for himself, he is on the downward road, wandering away from the higher intellectual light that ought to illumine his path. The little that the animals have learned and inherited after ages of experience and the little that man learned about the laws of nature during his unintellectual ages of savagery and barbarism are as nothing compared with that which the past century of civilization has revealed to the man of the present intellectual age.

There is a physical basis for all meteorological phenomena. There are laws of mechanics and heat that apply to the atmosphere, and as fast as we acquire the ability to discover these and reason out their consequences we shall perceive that law and order prevail in all the complex phenomena of the weather and the climate. Those who, for want of previous training, do not easily follow this method of presenting the problem will at least recognize that its fundamental principles are correct and that the methods of study are appropriate. We hope that the following sketch will stimulate some to take up our problems in earnest and devise either graphical, analytical, or numerical methods of resolving special questions.

I think all will agree that if we are to predict a rain or snow from fundamental premises by a rational deductive process, we must be able to first predict the temperature, moisture, and movement of the air. This is true whether we have in mind daily weather predictions or seasonal forecasts. The prediction of the wind, however, itself implies a knowledge of the distribution of the density of the air, and this latter is dependant upon temperature, moisture, and baro-

metric pressure, to say nothing of the rotation of the earth on its axis and the irregularities of continents and oceans.

Some have attempted making general predictions as to the character of the weather for a few days in advance by relying primarily upon the observed distribution of large areas of high and low pressure. I was myself obliged to do this during 1869 and again in 1871 and 1872, when I did the work of weather prediction for the Weather Bureau of the Army Signal Office. A most extensive study of types was carried out by Prof. Thomas Russell in the year 1888-1893. A specimen of his results is published in his meteorology, and confirms the conclusion that prediction by types is of extremely limited value in weather forecasts. When it comes to the prediction of a season, months in advance, one must look further and deeper. First of all, it is absolutely necessary to take into consideration the condition of affairs over the whole earth; second, it is equally necessary to consider the conditions prevailing up to a very considerable height in the atmosphere; third, we must apply physical laws and not empirical rules.

Of course a full knowledge of the actual physical condition of the atmosphere at any initial moment is not now practically attainable, but we all realize our needs in this regard, and international cooperation is being invoked to accomplish that which no one nation can do alone. Assuming that the time will come when we shall have the necessary observational data, it is now our duty to prepare the way for the complete utilization of the information that will eventually be at hand. We must therefore attack the physical, mechanical, and mathematical problems relating to the general and the special circulations of the atmosphere upon the real globe as it now exists, including in our consideration the fields of snow and ice, the mountain ranges, the continents and the oceans, and not forgetting the distribution of clouds. Hitherto the problem of the general circulation has been attacked by simplifying the fundamental assumed conditions. Thus the globe has been assumed to be without friction or other resistances and of uniform perfect spherical shape; the atmosphere has been assumed to be dry and, therefore, cloudless. The thermodynamic and the hydrodynamic portions of the problem have been violently separated from each other. It is true that there are great analytical difficulties to be overcome in the application of pure mathematics to this complex problem of nature. All must recognize that the solutions of the simplified problems given us by Ferrel, Erman, Goldberg, Mohn, Sprung, Helmholtz, Marchi, Willy Wien, Oberbeck, Margules, Pockels, Moeller, Diro Kitao, and others are so far from being applicable to the preparation of long-range forecasts that we can only consider them as helpful stimuli to those who are ambitious of engaging in work upon the more difficult problems that the real atmosphere presents.

I am emboldened to state the problem in all its complexity because, on the one hand the progress already made must greatly encourage the investigator, and on the other hand the consideration of the experience of the meteorological office in India demonstrates to my mind that the problems of long-range forecasting will yield to studies based on the broadest possible theoretical and physical treatment.

We will for a moment consider, especially, the condition of affairs in India, since one of the meteorologists of that country has recently applied to us to know whether anything of theory or experience is known in America that will contribute to elucidate their phenomena.

The principal problem presented to the Indian meteorologists is the prediction of the quantity and locality and, to a certain extent, the time at which the seasonal rains, known as the southwest monsoon rain and the northeast monsoon rain, will occur, especially the former. The accompanying 15 charts [not reproduced] show, by the red shaded areas, the regions in which notable droughts have prevailed since that of

1769, in so far as they are described in the "memorandum" of the Indian meteorological office for 1900.² The disastrous results of these droughts is well known to all. Sometimes millions of natives starve to death. Undoubtedly the terrors of a famine in India are aggravated by the peculiar sociological institutions of that country, especially the system of caste which often forbids one set of men to help their suffering brethren of another set. Thus it happens that the destitution and suffering incident to a failure of rain and crops is necessarily enormous, and produces a severe drain upon the resources of the British Government, and those of the numerous philanthropic and missionary societies that are seeking to elevate the general standard of civilization in India.

Studying the problem from a meteorological point of view only, we notice, first of all, that the area covered by the droughts of 1896 and 1899, and, of course, the resulting famines of the following years, were much larger than in any preceding year since 1769. This is not to be regarded as due to any effect of deforestation, or any great increase in the population, or as locally produced by a drying up of the soil, due to several years of light rain. The tinted areas on these charts indicate an actual local temporary diminution of the rainfall, and with that, of course, a diminution of cloudiness and an increase in insolation or sunshine, as well as in the temperature and dryness of the air near the earth's surface. The evaporation from the apparently dry ground during such droughts may indeed be much greater than the evaporation during a rainy season of normal cloudiness. These variations of cloudiness and rainfall are principally and primarily due to variations in the direction, intensity, and moisture of the so-called southwest monsoon, and to changes in the horizontal breadth of the stream of air flowing from the southwest. Therefore, the prediction of drought in India has been considered to be primarily a question of the prediction of the character of the southwest monsoon wind. I do not see how we can avoid the conclusion that a long-range prediction of the character of the seasons in any portion of the United States must also be fundamentally a prediction of the character of the winds that will blow over that region. The mountain ranges that force the winds to rise and flow over them only to cool down as they rise and deposit their moisture as rain or snow, remain the same year after year, but the winds themselves and the amount of moisture in the air change continually. The first seasonal predictions of the monsoon rains

were made for India in 1888, and we are warranted in saying that during thirteen years the only real failure has been that of the prediction of the monsoon season of 1899, which was the year of phenomenally great drought in that country. Of course, this failure has been a powerful stimulus to the study of methods by which to secure better results. The official forecasters of India relied necessarily at first, largely upon purely local conditions. Every condition that could be considered likely to diminish the force of the southwest monsoon was carefully studied by them. It was known that this monsoon is essentially an indraught of cool air from the Bay of Bengal and the Arabian Sea toward the heated plateau of Thibet. If, therefore, considerable snow still rested on the Himalayas and the regions northwest and northeast of India as late as March or April, this presumably would correspondingly diminish the heating effect of the sun in the springtime, and by so much diminish the indraught and the resulting monsoon rains. On the other hand, it was recognized that if the equatorial regions south of India, or even in the South Indian Ocean, were unusually warm or cold, this condition would also diminish or increase, respectively, the push of the air northward and affect the resulting monsoon. But the weather over these oceanic regions was at that time imperfectly known to the authorities at Calcutta, and it is not surprising that in the absence of this important information the predictions based only upon the condition of the air over the Asiatic Continent should occasionally go wrong. The need of studying the southern ocean was felt very early, and, finally, the funds were provided for compiling a daily chart of the monsoon area from the Himalayas, on the north, to the southern limit of navigation in the Indian Ocean at thirty-five degrees south latitude. These charts soon demonstrated that the southwest monsoon of India comes ultimately from the southeast trade winds of the South Indian Ocean. These latter winds are disturbed from their normal direction by the action of the heated continental area of Asia, Europe, and Africa, and not by that of Thibet alone. So great is the influence of the temperature and elevation, shape and extent of this Eastern Hemisphere that if we wish to study the meteorology of the globe on a proper basis, we must relinquish the assumption that the equator is the dividing line between symmetrical systems of wind in the northern and southern hemispheres. When we have adopted this radical change in the older ideas as to the general circulation of the atmosphere, we begin to be able to understand how the condition of affairs in the South Indian Ocean in March affects the condition in India in July and August, and in Japan in September; it may affect Australia, on the one hand, and the mountainous region of Central Africa, on the other, during the whole winter or rainy season of the Southern Hemisphere. In fact, the great disturbance that starts annually in the Orient seems to me to work eastward and northward, and in the course of one, two, or three years comes to be felt both in North America and in Europe.

Although the distribution of heat is the controlling factor in starting, changing, and maintaining the motions of the atmosphere, yet it is the rotation of the rugged earth on its axis that is the tremendous influence. The latter carries the air swiftly eastward, the former modifies these movements by maintaining a slow north and south interchange. It follows that the general problem of the atmospheric circulation is a discussion of the interference of these two fundamental influences. The oceans, continents, and mountains first modify the temperature of the air, and then modify its resulting motions. To appreciate the influence of the heat and vapor, or of the land and water, one must study an equal surface projection of the land hemisphere and the water hemisphere; but in order to appreciate the influence of the diurnal rotation of the globe, one must study an equal surface projection of the

²The following table, copied from page 32 of the memorandum on snowfall, dated June 5, 1900, as published by Mr. John Eliot, Meteorological Reporter to the Government of India, shows the location of the areas of severe drought during the past hundred and fifty years and renders the publication of the charts unnecessary:

Year.	Region of drought and area of subsequent famine.	Sq. miles.
1769.....	Bengal.....	100,000?
1791.....	Bombay, Hyderabad and Madras.....	200,000
1804.....	South Hyderabad and Deccan.....	150,000
1803.....	Ceded province of northwest Provinces and Central India....	100,000
1834.....	Bombay.....	80,000
1837.....	Northwestern Provinces, eastern estates of Rajputana and southeastern parts of the Punjab.	80,000
1838.....	Gujarat, Cutch and Kathiawar.....	50,000
1860.....	Part of northwest Provinces, Punjab and neighboring states of Rajputana.	55,000
1885.....	Northern part of Madras, South Hyderabad, north part of Mysore, south Mahratta districts of Bombay, Orissa, Bihar and all western Bengal.	150,000
1868.....	Rajputana, trans-Jumna districts of northwest Provinces, north and southeast districts of central Provinces, Punjab from Jumna to Indus.	200,000
1873.....	North Bihar, part of northwest Provinces and Oudh.....	300,000
1876.....	Madras and Deccan, Mysore and south part of Hyderabad....	55,000
1877.....	Central Provinces, northwest Provinces and Punjab.....	250,000
1896.....	Northwest Provinces, central Provinces, Central India and Rajputana.	to 300,000
1899.....	South Punjab, Rajputana, Central India, Berar, central Provinces, Hyderabad, Bombay Presidency and parts of Orissa, Chota, Nagpur and Madras.	to 400,000
		600,000
		to 700,000

northern and southern hemispheres. Both of these projections may be so combined in one as to show not only the distribution of temperature and moisture and the diurnal rotation, but also the distribution of the great resistances to the motion of the atmosphere offered by mountains and land as compared with the ocean. The isotherms, the lines of equal vapor tension, and the lines of equal pressure, may be combined into one set of lines of equal density of the air, which lines I have called *isostaths*, or lines of equal buoyancy, when proper allowance is made for the variations of gravity.³

The resistances to movement, which are often summed up in the word friction, are of several kinds, namely, first and least important, the viscosity of the atmosphere; second and more important, the mechanical obstructions offered by any flat plain or rising mass of land or even by the surfaces of waves of water; third and most important, the resistance often called friction, but which I have called convectional, which term may be explained as follows. Slowly moving masses of lighter air are, during the daytime, perpetually rising from the surfaces of land and water, while swiftly moving upper masses of potentially denser air are descending to replace them; moreover, the wind that strikes an obstructing shore or mountain range, is driven upward with a less horizontal velocity, while somewhere else the upper air must descend to replace it. In this perpetual interchange of upper and lower currents or vertical convection, the upper layers, which are generally moving the faster, are slowed down by giving a portion of their momentum to the sluggish masses that rise from below, while that portion of the rapidly moving upper currents that descends is forced to lose a portion of its momentum when it comes in contact with the ground, or when it pushes along the sluggish lower layers, making them move faster. There is, therefore, by virtue of this convection, not only a redistribution but even a loss of kinetic energy over the whole globe, and the effect is greatest when and where the convection is greatest. Of course this loss of momentum is simply a case of transformation of energy, since atmospheric pressures and temperatures are thereby increased. Moreover, the appreciable portion of this lost atmospheric energy that is transmitted to the surface waters of oceans and lakes reappears in ocean currents and in changes in the level or gradient of the ocean surface.⁴ The diurnal variation in insolation produces a diurnal change in convectional friction which in turn produces a diurnal change in the conversion of the kinetic energy of the wind into static or elastic atmospheric pressure, and to this I have for many years past been accustomed to attribute a large part of the observed diurnal variation of the barometric pressure. A corresponding interchange of momentum and pressure also accompanies plane horizontal motion over smooth water and low-lying rough land or even rough water. The atmospheric motions disturb the rotating earth and probably contribute to the variation of astronomical latitude and longitude discovered by Dr. Chandler.

The atmospheric temperature, or more properly its heat energy or thermal content, is subject to a steady loss by radiation outward to space, by radiation downward to the land and water, and by the chemical changes involved in melting snow and ice, in evaporation, and in the growth of plants and animals. The details of all this are complex and often obscure, but in general the thermal energy present in the atmosphere, or the excess of insolation over radiation, maintains a fairly steady condition of movement in the atmosphere for any given day of the year and probably, on the

average, for a whole season or year. An excess of heat causes inequalities in this steady condition which, from the point of view of analytical mechanics, are represented by the large and small but always temporary evanescent whirls, cyclones, or whirlwinds that course over the globe. These whirls must be treated as an essential part of the circulation of the atmosphere, quite as much so as are the resistances introduced by the stationary features of the earth's surface and the floating irregularities introduced by the moving clouds. Some method in dynamics must be devised to study the influence of these moving clouds and whirls. They are to be considered as a consequence of and superposed upon the irregularities introduced by areas of snow, land, and water, and by the annual apparent movement of the sun from the southern Tropic of Capricorn on December 21 to the northern Tropic of Cancer on June 20 and back again.

Our problems can not be resolved by merely examining charts of the globe, charts of the diurnal insolation, or charts of the prevailing upper and lower temperatures, moistures, and pressures, or charts of the orographic resistances to the motion of the atmosphere. The methods most appropriate for attacking the problems are not those of types and averages, but those of analytical mechanics, and it is only after the problems have been fully stated analytically and algebraically that we are likely to succeed in devising auxiliary geometrical and graphical methods of solving them that may eventually be made the basis of a system easily handled by practical experts. Of course such a system never can become a "popular method" capable of being used by every citizen, as some would seem to expect. Meteorology, like astronomy, is passing from the hands of amateurs into those of special students who can devote a lifetime to the work of resolving her riddles. Although I shall only suggest modes of solution, yet it will not seem trite if I call attention to the formulæ that present the mechanical conditions that must be fulfilled by every particle of the earth's atmosphere. These need not seem abstruse or profound because they are clothed in the simple algebraic language that is used in order to give precision; the fundamental principles expressed by these formulæ are easily comprehended by all. In general, it is not the principles that underlie modern science, but the art of reasoning upon these principles so as to obtain correct results, that constitutes our great difficulty. The healthy human mind is adapted to the easy apprehension of a simple statement of facts or principles, but there are relatively few who are gifted with the special powers of logical intuition and argumentation needed in the simultaneous consideration of many principles. The most complex machine is merely a combination of simple elements. The beauty of geometry is exhibited in its simplicity. The complex atmospheric conditions to be considered in meteorology render its problems the most difficult of solution of all that occur in science.

I.

Historically speaking, the first of the fundamental laws to which the air was found to be subject was its expansion by heat as shown by Galileo's air thermoscope, and next came Boyle's law of elastic tension. Both combined constitute the law that expresses the possibility of the existence of a gas as distinguished from a liquid or solid; it is expressed by the equation of elastic pressure or the equation for the gaseous condition. The elastic pressure at any point in the atmosphere acts in all directions equally at that point. A unit mass of air will occupy a unit volume of space when it is subjected to a special temperature and pressure. A kilogram of ordinary pure dry air will occupy 1/1.29305 cubic meter of space when its temperature is 0° C., and when its elastic pressure is that of the so-called standard atmosphere, or that cor-

³ See Annual Report of Chief Signal Officer. 1889. Part II. Preparatory studies for deductive methods.

⁴ Perhaps the losses by east winds and by west winds counterbalance each other, so that there is no appreciable residual influence on the speed of the rotation of the solid earth itself.

responding to the weight of a column of mercury 760 millimeters high, having a temperature 0° C. and pulled downward by the normal force of gravity at sea level and latitude 45°, where $g_0 = 980$ centimeters per second. For any other pressure, p , and temperature, t , on the centigrade scale (or T on the absolute scale, $T = 273^\circ \text{C.} + t$) this volume will vary as is expressed by the formula—

$$pv = RT \text{ or } v = \frac{RT}{p}. \text{ Or } \frac{1}{v} = \text{density} = \frac{p}{RT} = \sigma.$$

The constant $R = 29.2713$ for ordinary pure dry air, and 47.060 for aqueous vapor. The kilogram, the meter, and the centigrade degree are the fundamental units.

This equation merely expresses the laws of Boyle and Mariotte, Charles, and Gay Lussac, to the effect that every perfect gas expands in proportion to its absolute temperature and in inverse proportion to its elastic pressure.

For the ordinary atmospheric mixture of air and vapor the barometric pressure is the sum of the elastic pressures of all the components, and the density is the sum of the densities of the components.

The more complex equation deduced by Van der Waals, and confirmed by laboratory experiment, may be substituted for the above if considered necessary.

II.

The second condition to be enumerated is the hypsometric condition that the pressure of the atmosphere at any place depends on the weight of the superincumbent mass of air, except in so far as this pressure may be altered by the motions of the atmosphere. Hence, it ordinarily follows that each small change in altitude, which we call the differential of h , produces a differential change of pressure or a dp , such that—

$$dp = -\sigma g dh$$

where σ is the density and g the force of gravity prevailing in the layer of air whose vertical thickness is dh . This equation simply expresses the fact that the change in pressure dp is equal to the change in weight, or is equal to the weight of the thin layer whose thickness is dh .

On the right-hand side of this equation we have three variables, namely, the density, the force of gravity, and the variable altitude; but as the density depends upon both the temperature, the pressure, and the moisture of the air (and indeed upon its chemical constitution also, if as is very probable, there is a systematic change in the proportions of the various gases, as we ascend above the earth's surface) therefore, these varying quantities must be introduced in terms of their dependence upon the altitude (h) before this equation can be integrated. Under the simplest possible assumptions, Laplace performed this integration, and obtained the formula generally known as Laplace's hypsometric formula, expressing a relation between altitude, pressure, temperature, moisture, and gravity. A very complete study of this subject by Angot, published in the year 1898, shows that neither the simple assumptions used by Laplace, nor any others that are available, are likely to give results sufficiently accurate to be acceptable to future meteorology, although they may be reasonably useful when the barometer is used for approximately determining the height of mountains. The meteorologist must, for accurate work, not assume but observe the upper and lower temperatures and moistures by means of balloons and kites (rather than on mountain tops). The results of the best work that has hitherto been done, as recently published by Assmann and Berson, show that we have many surprises in store for us. More detailed and accurate explorations of the upper air must be made before we can with confidence express the relation between temperature (t), moisture (e), pressure (p), and altitude (h). Undoubtedly

these relations vary with time (τ), i. e., the hour of the day and the day of the year, with the geographical latitude (β) and longitude (λ) of the place, and especially with the location of the place relative to the oceans and the continents. We must, therefore, express these relations by a general analytical formula, namely:

$$\begin{aligned} t &= F' (\lambda \beta h \tau) \\ e &= F'' (\lambda \beta h \tau) \\ g &= F''' (\lambda \beta h) \end{aligned}$$

(a) As concerns gravity, although its changes are small, they are appreciable in studying the motions of the atmosphere, and when a proper gravity survey has been made over the continents and the oceans we shall obtain a satisfactory expression for the function F''' . At present Hellmann's is the best that we have.

(b) As regards the temperature, its irregular horizontal distribution over the actual surface of the earth must be reduced to an ideal sea level and expressed graphically by means of isotherms, after which these must be expressed algebraically by means of spherical harmonics. These functions were first applied to the distribution of terrestrial magnetism by Gauss, Weber, and Erman, and more recently used by Dr. G. W. Hill to express the distribution of the force of gravity at and above the earth's surface. Schoch has computed the coefficients of the terms that occur in the development of this function so as to satisfy 72 temperatures read off from Dove's isotherms for 1852. His formula reproduces the observed temperatures to within 1° Reaumur, and although the Reaumur scale is not now in use, yet I will quote the formula as given in Reaumur degrees in Dr. Frank Waldo's translation of the inaugural dissertation of Dr. Schoch:

$$\begin{aligned} \varphi &= \text{longitude from Greenwich.} \\ \theta &= \text{North polar distance} = 90^\circ - \beta. \\ t &= \text{temperature in degrees Reaumur.} \\ t &= 11.99^\circ + 0.13^\circ \cos \theta - 31.71^\circ (\cos^2 \theta - \frac{1}{3}) \\ &\quad + \sin \theta \{ [0.38^\circ + 2.66^\circ \cos \theta] \cos \varphi \\ &\quad + [0.23^\circ - 0.50^\circ \cos \theta + 0.29^\circ (\cos^2 \theta - \frac{1}{3})] \sin \varphi \} \\ &\quad + \sin^2 \theta \{ [0.23^\circ + 2.49^\circ \cos \theta] \cos 2\varphi \\ &\quad + [0.38^\circ - 1.40^\circ \cos \theta] \sin 2\varphi \} \\ &\quad - \sin^3 \theta \{ 0.80^\circ \cos 3\varphi \}. \end{aligned}$$

This equation therefore expresses that part of the function, F' , that depends upon the latitude and longitude. The portion depending upon the altitude is still to be determined more accurately from observations with balloons and kites, but we may consider the adiabatic law of convective equilibrium as an excellent first approximation. The portion of the function, F'' , depending upon the time (τ) is of a double nature. Exterior to the earth we have a great source of heat, the sun, whose intrinsic radiation may be assumed to be constant or variable according as we follow one or the other authority in solar physics. It is probable that the variations rarely exceed 1 per cent, but even this would be an appreciable quantity in dynamic meteorology. As we do not wish to complicate our atmospheric problem unnecessarily, we may assume that the solar thermal constant is really constant during the day, month, or year that we may happen to be investigating and that its value lies somewhere between 3 and 4 calories; that is to say, that at the mean distance of the earth from the sun, a surface of 1 square centimeter exposed normally to the solar rays, and outside of the earth's atmosphere, will receive in one minute three or four of the absolute units of heat known as a small calorie, or the amount of heat required to warm a gram of water 1° C.

But although the solar radiation may be constant, yet the distribution of that radiation on the earth's surface or within the earth's atmosphere is by no means constant. The so-called insolation at any moment of time depends on the altitude of the sun above the horizon, and on the intrinsic

transparency of the atmosphere above the observer, taking into full consideration the effect of cloud, haze, and fog. It is this variable insolation that largely determines the temperature in any part of the atmosphere and that introduces the element of time into the function, F' ; furthermore, the horizontal motions of the atmosphere, by carrying the heat to and fro, the ascending and descending motions by causing the cooling due to expansion and the warming due to compression, the formation of cloud and rain by causing new warm and cold spots, all introduce the element of time and a new set of inequalities in temperature and moisture as a result of motions that were themselves the result of the initial inequalities. Therefore, the temperature (t) and moisture (e) or the functions F' and F'' contain the time (τ) both directly and implicitly, so that the final functions represent the so-called dynamic equilibrium by reason of which the initial thermal irregularities are so modified as to form a new distribution of temperature, such that it shall be as nearly as possible consistent with what is called a steady system of motions in the atmosphere.

The distribution of direct insolation was long since investigated by Lambert, and in 1855 by L. W. Meech; an analysis of the subject, convenient for use in our dynamic equations, was given by Ferrel in his Professional Papers No. XIII and in his Researches and, subsequently, in another form by Angot who showed algebraically, numerically, and graphically the quantity of heat that must be received from the sun by a horizontal surface at various heights in the atmosphere, namely at the outer limit where the atmospheric absorption is zero, at a lower stage where the zenithal absorption is 0.1, and so on by tenths down to the surface where the absorption is 0.8. These tables are perfectly general and can be applied to any condition of the atmosphere at any time and place.

(c) We have spoken fully of the elements of the function, F' , that represents the temperature. Similar remarks may be made with regard to moisture; the function, F'' , must depend directly on the temperature, and the evaporation from the ocean, the lakes, the snow covered ground, the moist soil or vegetation, and must depend implicitly again on the winds. Concerning all of these details we have as yet relatively little satisfactory knowledge, and for the present we must be content to introduce the best approximations, or those that are most convenient for mathematical treatment.

(d) We now come to a physical question about which there is still much discussion, which of course illustrates again the uncertainty of our present knowledge. The solar radiation is a complex influence, and can produce electrical, thermal, optical, and chemical effects according to the nature of the substance on which it acts and the attendant conditions. A part of this radiation is absorbed long before it reaches the earth by what is known as the solar atmosphere. An appreciable part is absorbed by the gases and vapors that are undoubtedly present in the upper air. Notwithstanding this sifting process enough reaches the upper surface of the clouds, the ground, and the ocean to keep the atmosphere of the illuminated half of the earth in such a state of commotion by day that twelve hours of darkness can scarcely allay it. An exhaustive treatment of the effects of insolation on the atmosphere would require us to consider what becomes of each wave length or each group of waves that emanates from the sun. This leads us to the analysis of the sunbeam by means of the spectroscope, the bolometer, and other forms of apparatus such as the various sunshine recorders that measure in some cases a heating effect, in others a chemical effect produced by the sunshine. Our first attention must be given to the heating effect, as it is probable that over nine-tenths of the radiant solar energy produces heat in our atmosphere either by direct absorption by the air or by conduction, radiation, and convection from

the earth after the earth's surface has absorbed it, or by conduction, radiation, and convection from the ocean after it has been absorbed by the ocean and is returned to the atmosphere in part as the latent heat of vapor of water. This class of problems admits of the most detailed study but may best be treated in a general statistical way. The latter alone is practicable for dynamic meteorologists, who will do well to follow very closely the path laid down by Dr. W. Zenker in his work, *The Distribution of Heat on the Earth's Surface*, Berlin, 1888, and *The Thermal Basis of Climates*, Halle, 1895. According to his figures, some of which will be recognized as based upon the old value of the coefficient of absorption of the unit thickness of the atmosphere, 0.778, we have the following table showing the total sum of the thermal effects of the rays absorbed directly by the atmosphere and absorbed after the diffuse reflection that goes on in the atmosphere and at the earth's surface.

The heat received by a horizontal unit surface in a unit time in terms of the normal unit of radiant heat outside the atmosphere (from Zenker's Distribution of Heat).

Z. D. of the sun.	Surface outside the atmosphere.	Surface below the atmosphere.	Absorbed by atmosphere.	Absorbed by earth.	Absorbed by sea.	Absorbed by snow.
0	1.0000	0.778	0.222	0.9980	0.8955	0.7715
10	0.9848	0.759	0.236	0.9117	0.8790	0.7577
20	0.9397	0.716	0.244	0.9685	0.8530	0.7301
30	0.8860	0.646	0.230	0.7918	0.7616	0.6558
40	0.7800	0.558	0.213	0.6911	0.6618	0.5748
50	0.6488	0.440	0.203	0.5688	0.5387	0.4723
60	0.5000	0.310	0.190	0.4287	0.3423	0.3521
70	0.3420	0.175	0.167	0.2698	0.2351	0.2320
80	0.1736	0.064	0.130	0.1110	0.0850	0.0914
90	0.0000	0.000	0.000	0.0000	0.0000	0.0000

From these fundamental figures Zenker computes the distribution of insolation for any part of the earth's surface, for a mean solar day and for the whole year, over the land, the ocean, and the snow covered surfaces, but into these details we do not now need to enter further.

(e) Besides this direct insolation our terrestrial temperatures are also affected by the reflected solar rays, reflected not only from visible surfaces on the earth and from the larger particles that form clouds, but also from the invisible particles of aqueous vapor and dust that float throughout the atmosphere. These exert a selective reflection; the finest vapors and perhaps even the molecules of the gaseous components according to Rayleigh send us the blue sky light, but the larger particles send us also the longer waves of heat or the warm portions of the spectrum. When the sky is of a deep blue tint the air is dry and comparatively little heat is reflected to us by the atmosphere, but when a haze over-spreads us, and is formed of small particles not too far apart, it reflects a very considerable amount of heat to the observer as a sum total of that sent from the whole apparent hemisphere of air above him. The amount and quality of this diffuse reflection has been studied by Wiener, Homén, and others in papers recently published. By utilizing these works we are now in a position to cover the globe with lines of equal insolation on which we build lines of equal absorption of heat by the earth's surface, whence follow lines of equal heating, of equal radiation outwards, and of equal temperature effect on the atmosphere and the globe so far as the initiative thermal action of the sun is concerned.

III.

There is another thermal condition to be considered, namely, the air must obey the laws of thermodynamics; in other words, the molecular energy contained within a mass of air is equal to the sum of its sensible heat and its latent heat, and in atmospheric processes the energy of the molecules is

frequently transformed, at least in part, into the energy of the whole mass or vice versa. When a quantity of heat, dQ , is added to or taken from a mass of free air, there may result changes of temperature or changes of volume, also evaporation or condensation of the vapor; if the air is confined in any way as by friction there may also be changes of pressure. The energy is not destroyed but is manifested in other ways. This law of the conservation of energy is the basis of the following fundamental equation in thermodynamics, which expresses the fact that an increase of temperature, dt , can be produced by the quantity of heat expressed by Cdt , and again that an increase of volume, dv , which means a little internal and much external work, can be produced by the quantity of heat, $A p dv$, and that if we add together these two quantities we get the total quantity of heat that is communicated to the mass of air. This thermodynamic condition is expressed in the following equation:

$$dQ = C_p dt + ART dv/v = C_p dt + A p dv$$

In general, therefore, this equation expresses in exact terms the principle that any change in the quantity of heat in a mass of air, whether caused by sunshine, by radiation, absorption, or conduction, or by any other process, must produce changes in temperature or volume, or both, that are related to each other by the condition thus algebraically expressed.

In addition to the operations by which heat is manifestly communicated to the air from the hot surface of the ground, or given up by the air to the ground and to outer space by radiation, there is another very important but more insidious source of heat. By evaporation from the surface of the moist land and leaves of plants, and especially from the surface of the lakes and oceans, the atmosphere receives an amount of moisture that must be on the average equivalent annually to the normal total snowfall and rainfall of the earth. In the equatorial regions this moisture enters the atmosphere at a temperature of 20° C. or higher, but in the polar regions it enters the atmosphere at temperatures that may be below freezing. Now the latent heat of melting and of evaporation and the specific heat of aqueous vapor are so large that the layer of water, averaging 3 feet in depth over the whole globe, that is thus annually added to and extracted from the atmosphere represents a transfer, an increase, and a subsequent loss of an immense quantity of heat. Wherever this invisible vapor is condensed into cloud, rain, hail, or snow a corresponding amount of latent heat becomes sensible, the air is locally warmed, and rapid motions take place in the atmosphere. This sensible heat is eventually lost by radiation, but, generally speaking, not before it has had a powerful influence on the motions of the atmosphere; indeed, it is the addition, the presence, and the loss of this heat, which is first latent and then sensible, that initiates most of our weather phenomena. The evaporation of water and the condensation of aqueous vapor must be considered as methods of adding heat to the air; on the other hand, evaporation of cloud, etc., cools the air; and these operations must proceed in accordance with the thermodynamic law above expressed.

In many cases atmospheric processes go on so quickly (as when air is rising rapidly, and especially in the interior of a cloud, where there can be no considerable radiation) that a given quantity of moist air retains practically all its heat while going through the process of formation of cloud and the subsequent dissipation of that cloud; if some is lost by radiation and some by the fall of rain, snow, or hail from the cloud to the ground, yet what occurs in an hour or a day can represent only a small percentage of the whole. Those physical changes or processes among molecules of matter that do not involve any change in the quantity of heat are called

adiabatic. In these cases the dQ of our equation is zero. The discussion of the adiabatic changes of temperature, moisture, volume, and altitude in a mass of moist air has been rendered quite simple by the graphic construction invented by Hertz and improved of late by Professor von Bezold and his pupil Dr. Otto Neuhoff. The problems may also be solved numerically, and perhaps more accurately but scarcely so perspicuously, by means of the extensive numerical tables prepared by our colleague, Professor Bigelow, and published in his report of International Cloud Observations, Annual Report Chief of the Weather Bureau, 1898-1900, Vol. II. From these tables or from the adiabatic diagrams we learn the temperature, moisture, and other conditions through which a given mass of air must pass when it is in motion, either horizontally or vertically, from any region to another in which different pressures prevail.

IV.

We now come to consider another condition that the whole atmosphere, as well as any portion of it, must fulfill when in motion. Owing to the fact that the air is almost perfectly elastic, and that it expands very rapidly into any adjacent empty space, it is impossible under ordinary meteorological conditions to make or maintain a vacuous space within the atmosphere. Again when a portion of the atmosphere is in motion the adjacent portions affected by this, and even distant portions, must have corresponding motions. Putting these two considerations together we say that the motions that may prevail throughout the atmosphere must be considered as continuous, just as we consider the mass of the atmosphere as continuous; that is to say, there are no breaks or gaps in the mass and no sudden independent arbitrary changes of motion. So far as the continuity of motion or velocity is concerned, it is expressed by the sign of integration, by the process or method of accomplishing this, and by the assumptions that are implied therein. So far as the continuity of mass is concerned, this condition is expressed by the so-called equation of continuity, which simply says that if the density of a fluid is uniform, then as much matter must go into a given space as flows out of it, or if the fluid is an elastic gas, then the change of density is the excess of gas crowding into a given space over the deficit made by the outflow from that space. If ρ is the density, τ the time, u , v , and w the velocities of motion of any particle in the direction of positive x , y , z , respectively, then the condition of continuity as to mass may be expressed by the following equation of partial differentials:

$$\frac{\partial \rho}{\partial \tau} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} + \frac{\partial (\rho w)}{\partial z} = 0$$

which equation simply says that at any point the rate of change of density with time is opposite to the changes in the products of the density by the three component velocities; or an increase in velocity diminishes the density.

V.

The law of the conservation of energy imposes another general condition on atmospheric phenomena. All motions and mechanical phenomena originating in the atmosphere, whether massive or molecular, must be consistent with the principle that the sum total of the kinetic energy, E_k , the potential energy, E_p , and the intrinsic energy, E_m , must be constant, or if the sum total changes with time (as, for instance, if possibly the sun changes his radiation and our insolation or supply of energy) then the law of such changes must be given explicitly as one of the conditions of the problem. The conversion from mass energy to molecular energy, or the giving up of either molecular or mass energy by the atmosphere

to the mass of the globe, or the loss by radiation of molecular energy that may just previously have been mass energy is all represented by a so-called dissipation function, F . If for any given boundary surface, S , we draw a normal inwardly into the atmosphere and this normal makes such angles with the coordinate axes that l, m, n are the respective cosines, u, v, w being the component velocities, and p the elastic pressure, then this equation of conservation of energy may be written as follows:

$$\frac{\partial (E_t + E_v + E_w)}{\partial \tau} + 2F = \iint p(lu + mv + nw) dS$$

VI.

We have now expressed first, the conditions relative to temperature, volume, and pressure, then the conditions that involve motion and continuity of motion, mass, and energy; but a most important condition still remains, namely, the relation that must subsist between the pressure and the motion, or rather the momentum and the energy. Motion must be considered as the result of pressure, each particle of the atmosphere sustaining a pressure due primarily to the weight of all above it. The force of gravitation, the additions of heat and moisture, the centrifugal force due to the rotation with the earth and the resistances of its surface may be considered as the external forces acting on the atmosphere while the pressure at any spot is the internal force acting upon a local mass of air. These forces may produce either uniform or accelerated direct linear motions and they may also produce rotation on the part of the local masses of air. The general condition that for every motion produced by a force there must be an equal and opposite reaction is expressed by the following system of equations:

$$\begin{aligned} -\frac{\partial P}{\partial x} - \frac{1}{\sigma} \frac{\partial p}{\partial x} &= \frac{\partial u}{\partial \tau} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ &\quad - \mu \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right] \\ -\frac{\partial P}{\partial y} - \frac{1}{\sigma} \frac{\partial p}{\partial y} &= \frac{\partial v}{\partial \tau} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ &\quad - \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right] \\ -\frac{\partial P}{\partial z} - \frac{1}{\sigma} \frac{\partial p}{\partial z} &= \frac{\partial w}{\partial \tau} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \\ &\quad - \mu \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] \end{aligned}$$

in which the left-hand terms present the action of the external forces, P , and the internal pressure, p , upon a unit volume of air whose variable density is σ . On the right-hand side, the first four terms express the fact that the change of force or pressure along a given axis of x or y or z , as expressed on the left hand, may produce changes in velocity along all three of the axes as well as independently along that axis itself. The last three terms, or those within the brackets that are multiplied by μ , express the retarding effect of the viscosity or internal friction, μ , that is small but appreciable in all gasses at ordinary temperatures. The terms depending on viscosity would have no influence if all the motions were absolutely uniform throughout the mass, that is to say if the second partial differentials of the velocities with respect to the coordinates of the point (x, y, z) were zero.

The effect of all the various forms of resistance and friction between the atmosphere and the earth, before described, does not enter explicitly into these equations, but must be considered in connection with the boundary conditions and initial conditions after the first integration has been effected. Owing to the irregularity of the resistances of the earth's

surface, a perfect integration would inevitably necessitate the use of complex functions; approximate solutions may be found by using spherical functions or graphic methods instead of analysis, but for a first approximation we may simply introduce the conception of the rough land and smooth water hemispheres, as we did in regard to the distribution of heat and moisture. If we neglect ordinary viscosity and sliding friction, we must at least not neglect convectional resistances and must introduce these latter as functions of the vertical distribution of temperature, since this determines the ascending and descending vertical currents that produce the mixture of slow and rapid horizontal currents whence result their mutual resistances or interferences and the conversion of kinetic into potential energy. The extent of the diurnal barometric variation, ± 0.10 inch, as compared with the depth of air affected, shows the importance of this item relative to the total energy of the wind. The convectional resistance due to mixtures produced by impact of currents against continents and mountains are of less importance and do not depend on vertical temperature gradients.

VII.

It is to be expected that long before we have attained to what may be called a complete solution of all these equations of condition we shall by means of general theorems have at least obtained some approximately correct ideas concerning the general mechanics of the atmosphere. Already many have proposed to skip this long process of reasoning and substitute for each locality and for the prospective rigorous solutions some approximate sine and cosine formula, as though it were certain that the Bessel-Fourier series or some other combination of periodicities would satisfactorily represent the motions and other phenomena of the atmosphere. But the problem is undoubtedly too complex for plane harmonics; we shall need to develop the original functions and integrals in a series of spherical harmonics or equivalent equations in which latitude, longitude, and altitude must play equal parts. A first step in this direction was taken by Oberbeck, who adopted a few terms of the harmonic series, but a second equally important step was taken by Margules, who, assuming a simple distribution of temperature over the earth's surface, worked out a system of tesseræ, dividing the globe up into regions of low pressure and high pressure with the attending winds. A more exact adherence to actual temperatures and especially a more complete consideration of the orographic resistances of the earth's surface will undoubtedly lead to more complex results.

The question as to whether there can be any long continued important natural periods as distinguished from forced periods in atmospheric phenomena, has been discussed by Herrmann in the Bulletin of the American Mathematical Society for June, 1896. Starting with an assumed initial temperature and a rate of revolution corresponding to any initial state of equilibrium, Herrmann shows that, in general, the atmosphere can not settle down to a new state of equilibrium except it be one of motion and dynamic equilibrium. There must be an interchange, not only between the poles and the equator but between the east and west. Then the question recurs, can these internal movements become perfectly steady or be so adjusted to each other and to external conditions that each system remains invariable within any given zone of latitude. Herrmann concludes that steady motions and stationary pressures can not exist in an atmosphere covering a homogeneous globe, and presumably the same result must follow for the actual, irregular globe. The motions of the real atmosphere must consist of irregularities and periodic oscillations superimposed upon more uniform regular progressions, but never repeating themselves. It also follows that the indiscriminate monthly and annual averages,

useful in climatology, are quite inappropriate and misleading from a dynamic point of view. Another phase of the same problem is found in the question: can simple seasonal variations in atmospheric pressure and wind be forced to exist by reason of regular seasonal variations in the temperature due, for example, to periodic variations of the solar constant, or to any other influence outside of the atmosphere? We infer that this also is not possible for a globe and atmosphere of the existing dimensions and physical properties, but might be for a much deeper atmosphere.

It would seem, therefore, that long-range predictions can best be carried on quite independent of any simple system of strictly periodic functions.

VIII.

So far as the prediction of rain is concerned, we still have to recall the fact that rains are due in some way or other to the rather rapid ascent of warm, moist air. In some cases, like the Indian mousoon rains, the air is forced to rise up by its attempt to pass over a mountain range. The analytical study of this problem has just recently been attempted by Prof. Dr. F. Pockels of the University of Heidelberg, whose memoir was published in the MONTHLY WEATHER REVIEW for April, 1901, with added notes in the REVIEW for July, 1901.

Professor Pockels has worked out an analytical solution of the equations of motion combined with those of thermodynamics for the case of a horizontal, deep stratum of moist air of uniform temperature and humidity, forced to ascend a mountain slope whose curvature can be represented by a simple sine and cosine formula. He shows the amount of precipitation that must occur (of course in the form of cloud) at each step of its ascent, and that within a given volume of moist air the greatest density of cloudy precipitation will occur in many cases before that volume has reached the highest point in its ascending path. He also shows that in many cases an upper stratum of air in the general moving mass will rise sufficiently to form condensation before the lowest stratum can do so. Pockels does not attempt to distinguish between the original cloudy condensation and the subsequent precipitation, as rain or snow, but assumes that in general they must be in proportion to each other. Of course it is well understood that the actual rainfall is but a small percentage of the total cloudy condensation. The course of reasoning that Pockels has carried out analytically can doubtless be expressed geometrically or graphically, as also by the method of numerical quadratures. He has therefore contributed greatly to the development of the whole series of approximate integrations that must be relied upon when we attempt to build up a systematic deductive treatment of our problems. His work shows that if by any process we can determine for a long time ahead what the temperature, moisture, and wind will be over a given region, we may then hope to attempt for that region a long-range prophecy of the rainfall. But he particularly calls attention to the fact that in making such use of his method one must not rely upon the simple average temperature, moisture, and wind, since if these vary considerably during the time under consideration, as they naturally do, then the most favorable conditions may give rain when the average conditions will not do so. In other words, the formation of cloud and rain is a discontinuous process, occurring and stopping at certain limiting conditions, and it is only the favorable conditions that we really need to consider. This important principle simplifies rather than complicates our problem, for all the ultimate phenomena in meteorology have this same sort of discontinuity, due to their intermittent character. For instance, the air does not always move in simple, smooth curves, but often stops and whirls around until a certain excess of

energy has been consumed and then goes forward for a time, until by reason of differential resistances or a new access of energy it is again in condition for another set of whirls; and so it goes on over and over, whirling when the moment of the push forward exceeds the moment of resistance or when the moment of rotation exceeds the moment of friction, but running along smoothly when it is less than the moment of friction. Closely analogous cases are presented in the two modes of flow of water through a tube, so skilfully shown experimentally by Osborne Reynolds, or in the lamellar flow of air between two horizontal plates, investigated analytically by Lord Kelvin. In fact, the movements of the lower atmosphere on a small scale and the movements of the whole atmosphere, when we take a comprehensive view of it, offer grand applications of the laws of turbulent and of tumultuous movement, developed by Boussinesq in his study of the theory of the flow of water in rivers and canals. (See my "Preliminary Studies for Deductive Methods in Meteorology," Report Chief Signal Officer, 1889.)

IX.

The initial conditions prevailing in the upper and lower air at any time offer a most important observational source of uncertainty in beginning our proposed long-range forecasts. The equations that have been given in the preceding paragraphs imply that we know the conditions prevailing overhead as accurately as we do those at the surface of the earth. But this of course is not and may never be the case. One might have hoped that long continued records from mountain tops, and the numerous balloon ascensions of the last century, might have given us some basis for conclusions as to the conditions prevailing a few miles above sea level. But the more careful recent work with self-recording apparatus, and especially that with the so-called sounding balloons, has shown that at least in Europe, conditions prevail at great heights that were wholly unexpected. The newer observations are in fact found to be nearly accordant with those indicated by theoretical thermodynamics and hydrodynamics, and it is believed that all the older observations, which gave results contrary to mechanical theory, were really affected by very large errors of observation. The newest results of observation and theory are fully set forth in a recent work by Assmann and Berson, *Ergebnisse, etc., Results of Observations of Scientific Balloon Voyages, Berlin, 1900*, in three volumes, large quarto. This work shows that there is quite as great a diversity in temperature and moisture, and perhaps also in pressure, ten miles above sea level as there is at the earth's surface. Until quite recently the tendency had been to assume that above the so-called "lower" air there exists a layer embracing quite an appreciable part of the atmosphere in which temperature, moisture, and the chemical or gaseous constituents, as also the dust, are all quite uniform and subject to slight changes, if any. But at present we are forced to recognize the presence of great variations of temperature, and even of a very considerable atmosphere of new gases that are unimportant in the lower atmosphere, as also the occasional presence of dust, meteoric dust, or volcanic ejecta, which may have a very great influence on the local air temperatures, because they absorb solar radiation by daytime and warm up the adjacent air by convection, but also radiate rapidly, and, therefore, at night cool the adjacent air by conduction. Woodward maintains that the tidal action of the sun and moon on this upper atmosphere, which is more like a ring of discrete particles than a gas, may produce appreciable periods in the meteorology of the lower air. I can but anticipate that in addition to the use of sounding balloons and kites, we may get some idea of the general condition of the air, or at least the lower $\frac{1}{10}$ of it, by observations of the scin-

tillation of the stars, of the blueness and polarization of the sky light, of the intensity of special dark bands seen in the solar spectrum, or of cold bands observed with the bolometer, or again, by determining the general diathermancy of the atmosphere by means of Angström's, or some other form of, pyrheliometer.

X.

Although our direct observations of upper atmospheric conditions in general are limited and imperfect, yet it happens that much may be accomplished whenever we are able to demonstrate the ascent of a mass of air to the upper regions and its descent to the surface in some distant part of the globe. In such cases we may know quite closely the initial surface conditions, and can therefore, by thermodynamics, calculate the intermediate and final conditions of the descending mass, no matter how high it has ascended above the ground. Unless it has been subjected to some influence external to the earth, we ought to be able to quite exactly foresee its condition when it returns to the surface. In such calculations the most difficult problem is to determine, or even to estimate, the change that takes place in the ascending mass of moist air (1) by virtue of the fall of rain or snow carrying both matter and heat out of the original air down to the ground and (2) by virtue of the radiation of heat from the gases, aqueous vapor, dust, and, especially, the liquid cloud particles or the solid ice crystals of the snowflakes. At the present time, indeed, we understand the first of these changes, but the second class can only be crudely estimated; they can not even be measured directly, but the accuracy of our estimates can be tested by the observed condition of the air before and after it descends to the ground. The average coefficient of radiation of dusty, moist air was first determined by Maurer.

As the formation of clouds is a fundamental phenomenon, affecting the motions of the atmosphere, on the one hand, and a necessary precedent to rain, on the other, therefore the long-range forecaster must consider the theorems worked out by Brillouin in his recent memoir, *Vents Contigus, etc.*, or the *Action of Contiguous Currents of Wind and the Mixture of Air in Forming Clouds*. It is well recognized that such clouds of mixture give but little rain and are rarely of importance to the agriculturist or hydraulic engineer. Therefore they do not directly enter into a long-range forecast of rainy seasons in their own locality; but, in so far as they form extensive layers, they screen the earth from the warmth of the sun and from the coolness due to outward radiation, and therefore may have a very important influence on the distribution of heat in the atmosphere, and therefore on the currents of air, and eventually on the rainfall itself in some distant region.

XI.

But when all that can be has been done to ascertain or determine the distribution of temperature or moisture in the atmosphere, there still exists the fundamental mathematical difficulty of our problem in the introduction of all this meteorological data into the equations of pressure and motion or energy given above in Section VI (on page 558) and the integration that must then necessarily be executed. Since we are dealing with the globe as a whole, we must convert these equations into those for polar or spherical coordinates, as was elegantly done by J. Cottier, *MONTHLY WEATHER REVIEW*, July, 1897. In the process of integration there is scarcely any term or force that can be neglected in general, although some may be negligible in special cases. Rigorous investigations are of course not to be thought of; the equa-

tions must be developed into series, as is done in the lunar theory and other complex astronomical problems, and the attention must first be concentrated upon the computation and tabulation of the important terms of the series.

Perhaps the most radical method of simplifying the problem and at the same time doing approximate justice to the influence of the reaction upon each other of the aqueous and continental hemispheres of the earth, is to substitute for the observed temperature, moisture, and pressure, not a development by spherical harmonic functions, but a general system of isobnormals symmetrical about the north pole of the land hemisphere and the south pole of the water hemisphere, locating these poles respectively at longitude 0° and north polar distance 38° , and at longitude 180° north polar distance 142° , thus making Greenwich both the meteorological and the orographic north pole. This will minimize the influence of South Africa and South America in the Southern Hemisphere, and it will give us a resulting system of winds representing fairly well the Asiatic southwest monsoon of summer and northeast monsoon of winter; it will show, for instance, that the air carried from the southern Indian Ocean into the Northern Hemisphere east of Asia in June and July must descend within the arctic circle and flow out of that eventually during the subsequent winter over North America. This flow will be found to be not a so-called steady movement, but one subject to great oscillations, east and west, from month to month; it explains the elliptical upper isobars shown in the *MONTHLY WEATHER REVIEW*, December, 1895, and November, 1896, Chart VII; also the climatic changes observed in India by J. Eliot, *Quar. Jour. Roy. Met. Soc.*, January 18, 1896; the *Variations in Storm Tracks, etc.* The general circulation tends to obey the law laid down by Ferrel for the currents over a globe that is perfectly frictionless and on which the lines of temperature, moisture, and pressure are symmetrical about the astronomical poles, namely, that all moments of inertia with reference to the earth's axis of rotation, which is also the atmospheric axis of equal density, must be symmetrical and must balance each other as to east-west and north-south motions. But in the case we are considering, of an unsymmetrical system of lines, this balance is produced, not by a careful adjustment of simple rectilinear or curvilinear motions and not by superposed movements, but by a combination of these with more complex systems of horizontal whirls, so that large oscillations are necessarily introduced. Here we find the general explanation of the recurring irregular variations in the climates, the seasons, and the annual mean average conditions over America and Europe, from what would otherwise be symmetrical, uniform, normal values. The influences of the Atlantic and the Pacific oceans on the continents immediately adjacent to them would be uniform from year to year were it not for the overpowering influence of the oscillations caused by this want of agreement between the geographic and meteorological poles or the geographical and meteorological equators.

This study has already in part, and will eventually in full, elucidate the connections that are sometimes temporary, at other times almost permanent, and in many cases reciprocally inverse, between the average condition of the seasons in distant parts of the globe. From the very earliest times the American colonists endeavored to elucidate the relations between the weather of the Atlantic States and that of Europe; the modern daily weather maps and monthly summaries show these relations to be distinctly due to the movement of the lower stratum of air, as when cold blizzards and high pressures move from northern Canada south to the Gulf of Mexico, or from northern Russia southwestward to the Mediterranean. John Eliot shows by the Indian monsoon charts that unusual rains on the east coast of Africa or in Madagascar, or even in the high mountains in the interior of Africa, are fol-

lowed by diminished rains in India. The researches of Rus-
sel (Director of the Observatory of Sydney) seem to show that
the droughts in Australia are accompanied or followed by
droughts in other portions of the world. The series of charts
compiled by Hildebrandsson, and published partly in his Re-
searches on the Centers of Action of the Atmosphere, show
that even when we adopt the arbitrary division of the year into
twelve months, and chart the monthly departures of pressure,
temperature, and rainfall from the normal values for those
months, there are certain broad generalizations that become
possible, and which would doubtless be still more pronounced
if more appropriate periods were adopted. Thus, the depart-
ures from pressure are greater in winter than in summer and
increase from the Equator toward the poles. The barometric
variations in the Azores and in the neighborhood of Iceland
are nearly always opposed to each other, and the curves for
Siberia and Alaska are generally inverse to each other. The
curves for Tahiti at the center of the South Pacific area of
high pressure, and those for Tierra del Fuego in the southern
temperate zone are inverse. Among all these changes there
appears, as yet, no law of successive transformations. By
plotting the barometric departures on charts of the world,
Hildebrandsson finds that the pressure is sometimes abnor-
mally high over the arctic region but low over the equatorial
region, or vice versa; at one time it is high over northern
Europe but low over the arctic and North America; at an-
other time it is low there and high here; during one month
it is high over the arctic and low over the antarctic, and an-
other month just the reverse.

The continuous set of daily maps of the Northern Hemis-
phere prepared by the United States Weather Bureau from
1875 to 1895, and actually published for nine years, have
often given us abundant illustrations of the fact that the
phenomena occurring in one part of the atmosphere are cer-
tain to affect other distant portions a few months later, and,
in fact, may continue to be felt even for years. If we recall
the observed movement of the vapor and dust ejected from
Krakatoa in 1883 (its spread at great elevations near the
Equator, moving westward around the world in a few days
and its very slow spread northward, reaching latitude 40°
north in nine months and 60° north in eighteen months), we
shall the more easily understand that even greater events
than this (such as those that produced the droughts of 1896
and 1899 in India), when once they occur, must propagate
their influences also slowly, but not necessarily in precisely
the same direction or manner. If there be a variation in
solar radiation, it must affect the land and water hemispheres
differently; our system of isotherms and isostats, that we
have idealized into curves symmetrical about the orographic
and meteorological axis of the globe, will be changed prima-
rily as to intensity, and perhaps eventually as to location,
and the general effect of solar variations on local climate will
thus become distinguishable.

By thus considering the land and water hemispheres of
our globe as the thermal and frictional disturbers of the
phenomena that would otherwise pertain to a uniform sur-
face, rapidly rotating and warmed symmetrically with refer-
ence to the north and south poles, and by introducing
convictional resistances instead of viscosity, we seem able
to take another step forward in meteorology and long-range
forecasting.

It is only by the study of these general phenomena and
their elucidation by the help of the laws of mechanics that
we can expect to realize satisfactory long-range seasonal fore-
casts. We shall arrive at the desired result sooner and better
by the study of the mechanics of the atmosphere than by the
search for elusive empiric periodicities, and it is in the hopes
of inducing some to turn their attention toward this study
that I have submitted these views.

RECENT PAPERS BEARING ON METEOROLOGY.

W. F. R. PHILLIPS, in charge of Library, etc.

The subjoined titles have been selected from the con-
tents of the periodicals and serials recently received in the
library of the Weather Bureau. The titles selected are of
papers or other communications bearing on meteorology or
cognate branches of science. This is not a complete index
of the meteorological contents of all the journals from which
it has been compiled; it shows only the articles that appear
to the compiler likely to be of particular interest in connec-
tion with the work of the Weather Bureau:

- American Journal of Science. New Haven. Vol. 12.*
Barus, C. Effect of Temperature and of Moisture on the Eman-
ation of Phosphorus, and on a Distinction in the Behavior of
Nuclei and of Ions. Pp. 327-347.
- The Astrophysical Journal. Chicago. Vol. 14.*
Pickering, E. C. Spectrum of Lightning. Pp. 367-369.
- Journal of the Western Society of Engineers. Chicago. Vol. 6.*
Wright, Wilbur. Some Aeronautical Experiments. Pp. 489-511.
- Nature. London. Vol. 65.*
Shaw, W. N. Meteorological Work for Science Schools. P. 128.
— The "Armor" Electro-Capillary Relay. Pp. 129-130.
— Fresh Light on the Antarctic. Pp. 153-155.
— The Inert Constituents of the Atmosphere. Pp. 161-164.
- Smith, D. T. Relative Velocity in Streams. P. 174.
- Science. New York. Vol. 14.*
Ward, R. DeC. Hail Prevention by Cannonading. [Abstract of
article by Friedrich Stengel.] P. 938.
Ward, R. DeC. The Dust Storm of March, 1901, and Glacial
Studies. [Abstract of article by Richter.] P. 938.
Ward, R. DeC. The Climatic Control of Government in the
Tropics. [Abstract of article by Alleyne Ireland.] P. 938.
Ward, R. DeC. Underground Temperature at Oxford. [Review
of article published by Radcliffe Observatory.] P. 938.
- Ward, R. DeC. Lehrbuch der Meteorologie. Von Dr. Julius
Hann. Pp. 966-967.
- Upton, Winslow. Physiological Effect of Diminished Air Pres-
sure. Pp. 1012-1013.
- Rotch, A. Lawrence. The Measurement of Wind at Sea. Vol.
15. Pp. 72-73.
- London, Edinburgh, and Dublin Philosophical Magazine. 6th series. Vol. 2.*
Jeans, J. H. and Newton, Isaac. The Theoretical Evaluation
of the Ratio of the Specific Heats of a Gas. Pp. 638-651.
- Popular Science Monthly. Lancaster. Vol. 60.*
Cox, John. Comets' Tails, the Corona and the Aurora Borealis.
Pp. 265-279.
- Proceedings of the Royal Institution of Great Britain. London. Vol. 16.*
Rayleigh, Lord. Flight. Pp. 233-234.
Marconi, G. Wireless Telegraphy. Pp. 247-257.
Dewar, J. Solid Hydrogen. Pp. 473-480.
- Scientific American. New York. Vol. 85.*
Collins, Frederick. The Slaby-Arco Portable Field Equipment
for Wireless Telegraphy. Pp. 425-426.
- Scientific American Supplement. New York. Vol. 52.*
Rotch, A. Lawrence. The Use of Kites to obtain Meteorological
Observations. Pp. 21718-21720.
- Symons's Meteorological Magazine. London. Vol. 36.*
— Hann's Text-Book on Meteorology. Pp. 177-179.
— Rainfall and Storms in November. Pp. 179-181.
— The Moon and Rainfall. Pp. 183-184.
— Weather and the Horns of the Moon. Pp. 184-185.
- Terrestrial Magnetism and Atmospheric Electricity. Baltimore. Vol. 6.*
Borgen, C. Report on the Magnetic Observations made during
the Total Solar Eclipse May 17-18, 1901, at the Magnetic Obser-
vatory, Wilhelmshaven, Germany. Pp. 167-168.
Eschenhagen, Max. Report on the Magnetic Observations made
during the Total Solar Eclipse, May 17-18, 1901, at the Magnetic
Observatory, Potsdam, Germany. Pp. 169-172.
Fraser, H. A. Denholm. Report on the Magnetic Observations
made at Dehra Dun, India, during the Total Solar Eclipse of
May 17-18, 1901. Pp. 173-176.
Moidrey, J. de. Report on the Magnetic Observations made at
the Magnetic Observatory at Zi-Ka-Wei, China, during the Total
Solar Eclipse of May 17-18, 1901. Pp. 177-178.
Farr, Coleridge. Report on the Magnetic Observations made in
Christchurch, New Zealand, during the Total Solar Eclipse, May
17-18, 1901. Pp. 179-180.
Hosmer, G. L. Report on the Observations of Magnetic Declina-
tion made at Sawah Loento, Sumatra, during Total Eclipse of
May 17-18, 1901, by the party of the Massachusetts Institute of
Technology, Boston. Pp. 181-183.
Claxton, T. F. Report on the Magnetic Observations made dur-