What makes an annular mode "annular"?

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ABSTRACT

Annular patterns with a high degree of zonal symmetry play a prominent role in the natural variability of the atmospheric circulation and its response to external forcing. But despite their apparent importance for understanding climate variability, the processes that give rise to their marked zonally symmetric components remain largely unclear.

Here the authors use simple stochastic models in conjunction with atmospheric model and observational analyses to explore the conditions under which annular patterns arise from Empirical Orthogonal Function (EOF) analysis of the flow. The results indicate that annular patterns arise not only from zonally coherent fluctuations in the circulation, i.e., "dynamical annularity", but also from zonally symmetric statistics of the circulation in the absence of zonally coherent fluctuations, i.e., "statistical annularity". It is argued that the distinction between dynamical and statistical annular patterns derived from EOF analysis can be inferred from the associated variance spectrum: larger differences in the variance explained by an annular EOF and successive EOFs generally indicate underlying dynamical annularity.

The authors provide a simple recipe for assessing the conditions that give rise to annular EOFs of the circulation. When applied to numerical models, the recipe indicates dynamical annularity in parameter regimes with strong feedbacks between the eddies and mean flow. When applied to observations, the recipe indicates that annular EOFs generally derive from statistical annularity of the flow in the middle latitude troposphere, but from dynamical annularity in both the stratosphere and the mid-high latitude Southern Hemisphere troposphere.

1. Introduction

"Annular" patterns of variability are structures dominated by their zonally symmetric components. They emerge as the leading empirical orthogonal functions (EOFs) of the Northern Hemisphere sea-level pressure field (Lorenz 1951; Kutzbach 1970; Wallace and Gutzler 1981; Trenberth and Paolino 1981; Thompson and Wallace 1998, 2000), the Southern Hemisphere zonal-wind and geopotential height fields (Kidson 1988; Karoly 1990; Hartmann and Lo 1998; Thompson and Wallace 2000; Lorenz and Hartmann 2001), the Southern Hemisphere eddy-kinetic field (Thompson and Woodworth 2014), the extratropical circulation in a hierarchy of numerical simulations of the atmospheric circulation (e.g., Robinson 1991; Yu and Hartmann 1993; Lee and Feldstein 1996; Shindell et al. 1999; Gerber and Vallis 2007), and aquaplanet simulations of the ocean circulation (Marshall et al. 2007). They are seemingly ubiquitous features in a range of geophysical flows.

Despite their ubiquity in the climate system, one key aspect of annular structures remains open to debate: What gives rise to their marked zonally symmetric components? Does the zonal symmetry of annular structures reflect coherent variations in climate across a range of longitudes? Or does it largely reflect the constraints of EOF analysis (e.g., Dommenget and Latif 2002; Gerber and Vallis 2005)? Consider a long-standing example: the so-called northern annular mode (NAM) emerges as the leading EOF of the NH sea-level pressure field (e.g., Thompson and Wallace 2000). It exhibits a high degree of zonal symmetry and its structure implies in-phase variability in climate between the North Atlantic and North Pacific sectors of the hemisphere. But as discussed extensively in earlier papers (e.g., Deser 2000; Ambaum et al. 2001), the two primary centers of action of the NAM do not exhibit robust correlations on month-to-month timescales. Does the annularity of the NAM arise from dynamic connections between widely separated longitudes that are simply masked by other forms of variability (e.g., Wallace and Thompson 2002)? Or does the annularity arise from the constraints of the EOF analysis (e.g., Dommenget and Latif 2002; Gerber and Vallis 2005)?

The purpose of this paper is to revisit the conditions that give rise to annular structures in the leading patterns of variability of the circulation. We will demonstrate that annular patterns can arise from two distinct characteristics of the flow: (i) "dynamical annularity", where variability in the circulation about its mean state exhibits in-phase correlations at all longitudes, and (ii) "statistical annularity", where the statistics of the flow (e.g., the variance, autocorrelation and spatial decorrelation scale) are similar at all longitudes. Both conditions can give rise to annular-like EOFs that make important contributions to the variability in the circulation. But only the former corresponds to coherent annular motions in the flow. Section 2 explores the impacts of "dynamical annularity" vs. "statistical annularity" on EOF analysis of output from two simple stochastic models. Section 3 provides theoretical context for interpreting the results of the simple models. Section 4 applies the insights gained from the simple models to the circulation of an idealized general circulation model and observations. Conclusions are provided in Section 5.

2. A Tale of Two Annular Modes

In the following, we will define *dynamical annularity* as the case where there are positive covariances between all longitudes around the globe, i.e.,

$$\mathrm{cov}_X(\lambda_1, \lambda_2) = \frac{\sum_{n=1}^{N} X(\lambda_1, t_n) X(\lambda_2, t_n)}{N} > 0 \qquad (1)$$

for all longitudes λ_1 and λ_2. With this notation, we take X to be a generic variable of interest (e.g., geopotential height or eddy kinetic energy), given as an anomaly from its climatological mean. If (1) is satisfied, there are coherent underlying motions which cause the circulation to vary in concert at all longitudes. We will define *statistical annularity* as the case where the statistics of the flow do not vary as a function of longitude, i.e.,

$$\mathrm{cov}_X(\lambda_1, \lambda_2) = f(\Delta\lambda). \qquad (2)$$

where $\Delta\lambda = |\lambda_1 - \lambda_2|$ is the absolute distance between the two points. This definition implies that the variance of the flow is uniform, i.e. $f(0)$, and the covariance between any two longitudes depends only on the distance between them, but not where the two points lie relative to the origin (prime meridian). These criteria are not mutually exclusive, and a flow could satisfy both criteria at once. One would only expect (2) to hold approximately in the presence of realistic boundary conditions, but in Section 4 we show the statistics of the observed atmosphere are remarkably annular, particularly in the Southern Hemisphere.

Here we illustrate how statistical annularity can give rise to an annular EOF, even in the case where there is no underlying dynamical annularity in the circulation (that is, the motions are explicitly local). We consider two 1-dimensional stochastic models, $X_1(\lambda, j)$ and $X_2(\lambda, j)$. The details of the models are given in the Appendix, but all the necessary statistics of the models are summarized in Fig. 1. In short, both models are random processes in longitude, are periodic over 360°, and have zonally uniform statistics (2). The distinction between the models lies in their covariance structures (Fig. 1c). For model X_1, there is explicitly no global correlation: variability at a given location is only correlated with other longitudes over a range of about ±90°. For model X_2 there is a global correlation of 0.1.

Note that since both models have zonally uniform statistics, the covariance structures shown in Fig. 1c are independent of the base longitude used in the calculations. Moreover, they contain all the information needed to characterize the EOFs of the two models; recall that EOFs correspond to the eigenvectors of the covariance matrix $c_{ij} = \mathrm{cov}_X(\lambda_i, \lambda_j)$. When the statistics are uniform, c_{ij} is simply a function of the distance between λ_i and λ_j, as illustrated in Fig. 1c.

The top three EOFs for the two models are shown in Fig. 2a and b. By construction (see discussion in the next section), both models exhibit exactly the same EOFs. The first EOF is perfectly annular, as the analytic formulation of the model allows us to take the limit of infinite sampling. As seen in Fig. 2c, the first EOF also explains exactly the same fraction of the variance in each model: 20%. The second and third EOFs characterize wavenumber 1 anomalies: all higher order EOFs come in sinusoidal pairs, increasing in wavenumber. The phase is arbitrary, as the two wavenumber 1 modes explain the same fraction of variance. For finite sampling, one would see slight mixing between the wavenumbers, but the top modes are well established, even for a reasonable number of samples.

The key result in Fig. 2 is that both models exhibit a robust "annular mode" as their leading EOF, and that both annular modes explain the same total fraction of the variance. Only one of the apparent "annular modes", however, reflects dynamical annularity in the flow.

From the perspective of EOFs, one can only distinguish the two models by examining their EOF spectra, i.e., the relative variance associated with all modes (Fig. 2c). By design, the annular modes (the leading EOFs) in both models explain the same fraction of the total variance (20%). The key differences between the EOF spectra from the two models lie in the relative variance explained by their higher order EOFs. In the case of model 1, the first EOF explains only slightly more variance than the second or third EOFs. In the case of model 2, there is a large gap between the first and second EOFs. It is the relative variance explained that provides insight into the relative importance of statistical vs. dynamical annularity in giving rise to an annular-like leading EOF.

The stochastic models considered in Figs. 1 and 2 highlight two key aspects of annular modes. First the models make clear that identical annular-like patterns can arise from two very different configurations of the circulation: (i) cases where the statistics of the flow are zonally uniform but the correlations are explicitly local (model 1) and (ii) cases with in-phase variability between remote longitudes (model 2). Second, the models make clear that the spectra of variance yields insight into the role of dynamical annularity in driving the leading EOF.

3. Theoretical Insight

For systems with statistical annularity, as in models X_1 and X_2, the EOFs can be entirely characterized based on the covariance structure $f(\Delta\lambda)$. Batchelor (1953) solved the EOF problem for cases with zonally uniform statistics in his analysis of homogeneous, isotropic turbulence in a triply periodic domain. Our discussion is the 1-D limit of this more comprehensive analysis. If the statistics are zonally uniform (i.e., homogeneous), then EOF analysis will yield a pure Fourier decomposition of the flow. All EOFs will come in degenerate pairs expressing the same fraction of variance, except for the single wavenumber 0 (annular) mode[1].

The ordering of the Fourier coefficients depends on the Fourier decomposition of f. The covariance function $f(\Delta\lambda)$ is defined for $0 \leq \Delta\lambda \leq \pi$, where we express longitude in radians. The variance associated with a mode of wavenumber k is then given by

$$\mathrm{var}(k) = \frac{1}{\pi}\int_0^{\pi} f(\lambda)\cos(k\lambda)\,d\lambda \tag{3}$$

For all k other than 0, there will be two modes, each characterizing this amount of variance.

Setting $k = 0$ in (3) shows that the integral of the autocorrelation function determines the strength of the annular mode. For systems with zonally uniform statistics, there is thus a nice interpretation of the strength of the annular mode: the fraction of the variance expressed by the annular mode is simply the "average" of the covariance function between a given base point and all other points. This will hold even in cases where the annular mode is not the first EOF.

Returning to the simple stochastic models in Section 2, we can now see how the two models were designed to have the same annular mode. The average value of the two covariance functions in Fig. 1c is 0.2 in both cases, so that the "annular mode" in each model explains

[1] For a discrete system with n points in longitude, the missing partner to the wavenumber 0 mode is one of the wavenumber $n/2$ modes which is degenerate, appearing constant on the discrete grid. But for any realistic geophysical flow the variance decays quickly for high wavenumbers and this mode is insignificant.

20% of the total variance. In model X_1, the average correlation of 0.2 derives solely from the strong positive correlation over half a hemisphere. That is, the annular mode is the most important EOF, but it only reflects the annularity of the statistics. In model X_2, half of the variance associated with the annular mode can be attributed to dynamical annularity, as given by the global baseline correlation of 0.1. The other half is attributable to the positive correlation on local scales, reflecting the spatial redness of the circulation.

Model X_2 shows that even in a system with dynamical annularity, the "strength" of the annular model is enhanced by the spatial redness of the flow, which exists independent of underlying dynamical annularity. The weaker spatial redness of the flow in model X_2 relative to X_1 is visibly apparent in the structure of its samples (compare Fig. 1a and b), while the presence of coherent dynamical annularity leads to the large gap between the fraction of variance associated with wavenumber 0 and other waves in the EOF spectrum in Fig. 1c. It follows that an annular EOF is more likely to reflect dynamical annularity when there is large separation between the variance explained by it and higher order modes. In this case, the "average correlation" over all longitudes arises from far field correlation, *not just* the spatial redness of the circulation.

The models in Section 2 are two examples from a family of stochastic systems with spatial correlation structure

$$f(\lambda) = (1-\beta)e^{-(\lambda/\alpha)^2} + \beta, \tag{4}$$

illustrated graphically in Fig. 3a. The parameter α is the spatial decorrelation scale (defined as the Gaussian width of the correlations in units of radians) and parameter β is the baseline annular correlation of the model. For systems with this spatial decorrelation structure, the leading EOF is always annular and the second and third EOFs always have wave 1 structure, even if there is no annular correlation (i.e., $\beta = 0$). This follows from the fact that a Fourier transform of a Gaussian is a Gaussian, such that power is always maximum at zero and decays with higher wavenumbers.

Fig. 3b summarize the variance explained by the leading EOFs of the system considered in Fig. 3a as a function of the spatial decorrelation scale (ordinate) and the amplitude of the baseline annular correlation (abscissa). The contours indicate the variance explained by the leading (annular) EOF; the shading indicates the ratio of the variance between the leading and second (wavenumber one) EOFs. Dark blue shading indicates regions where the EOFs are degenerate (explain the same amount of variance). White shading indicates regions where the first EOF explains about twice the variance of the second EOF.

At the origin of the plot ($\alpha \to 0$ and $\beta = 0$), the system approaches the white noise limit, and all EOFs become degenerate. Traveling right along the x-axis from the origin (i.e., keeping the spatial decorrelation scale α infinitesimally small and increasing the baseline annular correlation with β), we find that the variance associated with the wavenumber 0 annular mode is simply given by the value of β. Here the spatial decorrelation scale collapses to a single longitude, so all higher modes are degenerate, and the strength of the annular mode derives entirely from dynamical annularity.

If one instead travels upward from the origin, allowing α to increase but keeping $\beta = 0$, the strength of the annular mode increases as well, despite their being no dynamical annularity. These are systems where the annular mode only reflects the *annularity of the statistics*, not *annularity of the motions*. As α gets increasingly large, positive correlations will develop at all longitudes by virtue of the fact that the spatial decorrelation scale is longer than a latitude circle. At this point, the spatial redness of the atmospheric motions gives rise to a baseline annular correlation due to the relatively short length of the latitude circle. When the spatial redness of the flow exceeds half of a latitude circle (0.5 on the ordinate axis), then the variance of the leading (annular) EOF explains $\sim$ twice the variance of the second (wavenumber one) EOF.

Model 1 sits in the blue shaded region along the ordinate (see blue circle in Fig. 3b), with a spatial decorrelation scale of approximately 0.23 radians. Model 2 (the red square) was designed to have baseline annular correlation of 0.1 (i.e., $\beta = 0.1$), but with an annular mode that express the same fraction of variance, requiring a local correlation $\alpha \approx 0.13$ radians.

The simple models considered in this and the previous section provide insight into the conditions that give rise to annular EOFs, and to the importance of the variance explained by the leading EOFs in distinguishing between statistical and dynamical annularity. In the following sections we apply these insights to output from a general circulation model and observations. In the case of complex geophysical flows with out-of-phase correlations between remote longitudes (i.e., teleconnections), one must consider not only the variance explained by the leading EOFs, but also the spatial correlation structure $f(\Delta\lambda)$.

4. The annularity of the circulation in models and reanalysis

How does the balance between dynamical vs. statistical annularity play out in general circulation models and observations? In this section, we apply the insights gained from the simple models to longitudinal variations of the atmospheric circulation at a single latitude, e.g., variations in sea level pressure or geopotential height at 50°S. We focus on a single latitude to provide a direct analogue to the simple one-dimensional stochastic models in previous sections, albeit a single latitude serves as a stiff test for annular behavior. The northern and southern annular mode patterns are based on EOF analysis of two-dimensional SLP or geopotential height fields, where spherical geometry naturally connects the circulation at all longitudes over the pole.

a. Annular variability in a dry dynamical core

We first consider a moisture free, 3-dimensional primitive equation model on the sphere, often referred to as a dry dynamical core. The model is run with a flat, uniform lower boundary, so that all the forcings are independent of longitude. Hence the circulation is statistically annular, making it an ideal starting point to connect with the theory outlined in the previous section.

The model is a spectral primitive equation model developed by the Geophysical Fluid Dynamics Laboratory (GFDL), run with triangular truncation 42 (T42) spectral resolution and 20 evenly spaced σ-levels in the vertical. It is forced with Held and Suarez (1994) "physics," a simple recipe for generating a realistic global circulation with minimal parameterization. Briefly, all diabatic processes are replaced by Newtonian relaxation of the temperature toward an analytic profile approximating an atmosphere in radiative-convective equilibrium, and interaction with the surface is approximated by Rayleigh friction in the lower atmosphere. The equilibrium temperature profile is independent of longitude and time, so there is no annual cycle.

A key parameter setting the structure of the equilibrium temperature profile is the temperature difference between the equator and pole, denoted $(\Delta T)_y$ by Held and Suarez (1994). As explored in a number of studies (e.g., Gerber and Vallis 2007; Simpson et al. 2010; Garfinkel et al. 2013), the strength of coupling between the zonal mean jet and baroclinic eddies is sensitive to the meridional structure of the equilibrium temperature profile. A weaker temperature gradient leads to stronger zonal coherence of the circulation and enhanced persistence of the annular mode. We use this sensitivity to contrast integrations with varying degrees of dynamical annularity.

The temperature difference $(\Delta T)_y$ strongly influences the climatology of the model, as illustrated by the near surface winds (blue curves) in Fig. 4, and can be compared with similar results based on ERA-Interim reanalysis in Fig. 6. The results are based on 10,000 day integrations, exclusive of a 500 day spin up. The default setting for $(\Delta T)_y$ is 60° C, and drives a fairly realistic equinoctial climatology with jets at 46° latitude in both hemispheres. With a weaker temperature gradient, $(\Delta T)_y = 40^\circ$ C, the jets weaken and shifts equatorward to approximately 38°.

The annular modes, defined as the first EOF of zonal mean SLP following Gerber et al. (2008) and Baldwin and Thompson (2009), are illustrated by the red curves in in Fig. 4. By definition, the positive phase of the model annular models is defined as low SLP over the polar regions and thus a poleward shift of the model jet. The centers of action of the model annular modes in sea level pressure vary between the two simulations, and are indicated by the vertical black lines. In the following, we focus our analyses on latitudes corresponding to the centers of action of the annular modes, contrasting it with similar analysis at their nodes.

The top row in Fig. 5 compares the spatial decorrelation structure of sea level pressure anomalies as a function of longitude at these three key latitudes. Results for the integration with weak and standard HS temperature gradients are indicated by blue and red colors, respectively. The bottom row shows the variances explained by the leading EOFs of SLP calculated along the same latitude bands (i.e., the EOFs are calculated as a function of longitude and time along the indicated latitude bands). We applied a 10 day low pass Lanczos filter (Duchon 1979) to the data before our analysis to reduce the influence of synoptic scale variability, but the results are qualitatively similar when based on daily or monthly mean data. To further reduce the sampling uncertainty, the autocorrelation functions were averaged over all longitudes and the EOF spectra were computed directly with equation (3). This has the effect of imposing zonally symmetric statistics, which would be the case with infinite sampling, and the results are virtually identical if we use the full fields for the calculations.

We focus first on the equatorward center of action of the annular mode (left column). The spatial decorrelation scale of SLP anomalies is approximately 60° longitude in both integrations (Fig. 5a). The east-west structure of the correlations reflects the scale of synoptic disturbances and wave trains emanating in both directions. The similarities between the spatial decorrelation scales reflects the fact that the deformation radius is similar in both runs. The most striking difference between the two runs lies in their baseline annular correlations. In the case of $(\Delta T)_y = 40$ the east-west structure of the correlations rides on top of a zonally uniform correlation of approximately 0.3. In the case of the model with

$(\Delta T)_y = 60$, there is a weaker baseline correlation of approximately 0.1.

The difference in the underlying annularity of the flow explains the differences in the variance spectra shown in Fig. 5d. In both model configurations, the leading EOFs are annular; higher order modes generally increase monotonically in wavenumber with the exception of waves 5 and 6 , which explain larger fractions of the variance that waves 3 and 4, consistent with the synoptic structure of the correlation functions. The distinction between the EOFs between the two model configurations lies in their variance spectra. In the case of $(\Delta T)_y = 40$, the annular mode explains more than four times the variance of the second EOF. In the case of $(\Delta T)_y = 60$, the annular mode explains about two times the variance of the second EOF.

The differences in the variance spectra for the two model configurations are consistent with the theoretical arguments outlined in the previous section. Both model configurations exhibit dynamical annularity, as evidenced by the fact the spatial correlations are > 0 at all longitudes. However, the dynamical annularity is much more pronounced for the $(\Delta T)_y = 40$ configuration, consistent with the larger ratio in variance explained between the first and second EOFs. The $(\Delta T)_y = 60$ configuration is reminiscent of the simple stochastic model X_2, where the leading EOF explains approximately 20% of the variance in the flow: half due to the dynamical annularity; half due to the spatial redness of the flow.

The annularity of flow is notably different along the node of the annular mode, which coincides roughly with the maximum in the climatological jet stream (Fig. 5b and e). The leading EOFs of SLP along the nodes of the annular modes are again annular, as is the case at the equatorward centers of action (not shown). But along this latitude, there is no apparent baseline annular correlation in either model configuration (Fig. 5b). Accordingly, the EOF variance spectra exhibit little distinction between the variance explained by the first and second EOFs. The enhanced dynamical annularity in the $(\Delta T)_y = 40$ case is thus associated only with vacillations of the jet stream's position, not fluctuations in its strength, which would be reflected by dynamical annularity in SLP at this latitude.

At the minimum of the annular mode pattern on the poleward flank of the jet stream, Fig. 5c and f, the relatively small size of the latitude circle leads to a strong baseline annular correlation and thus clear dominance of the annular mode in the variance spectra. The spherical effect is more pronounced for the $(\Delta T)_y = 60$ case since the minimum in the EOF pattern is located very close to the pole (Fig. 4). As the length of the latitude circle approaches the scale of the deformation radius, a single synoptic scale disturbance connects all longitudes, enforcing zonally uniform statistics. While the result appears trivial in this light, this geometric effect may play a significant role in helping the annular mode rise above other modes in two-dimensional EOF analysis. The flow is naturally zonally coherent near the pole, and the tendency for anticorrelation between pressures at polar and middle latitudes may play a role in generating annular-scale motions at lower latitudes (e.g., Ambaum et al. 2001; Gerber and Vallis 2005).

Its important to note that the circulation is more realistic with the default Held and Suarez (1994) setting of $(\Delta T)_y = 60$, where the flow exhibits relatively modest zonal coherence. The stronger dynamical annularity in the $(\Delta T)_y = 40$ configuration is due to the weak baroclinicity of the jet and the zonally uniform boundary conditions. When zonal asymmetries are introduced to the model, the uniform motions are much reduced, even with weak temperature forcing (Gerber and Vallis 2007). Zonal asymmetries on Earth will thus likely both reduce the strength of globally coherent motions in the sense of equation (1), and break the assumption of uniform statistics in the sense of equation (2). We find, however, that both dynamical and statistical annularity are highly relevant to flow in reanalysis, at least in the Southern Hemisphere.

b. Annular variability in reanalysis

The data used in this section are derived from the European Center for Medium Range Weather Forecasting (ECMWF) Interim Reanalysis (ERA-I; Dee and coauthors 2011) over the period 1979 to 2013. All results are based on anomalies, where the annual cycle is defined as the long-term mean over the entire 35 year period. As done for the dynamical core, a 10 day low pass filter is applied to all data before computing correlations and performing the EOF analyses. Note that qualitatively similar results are derived from daily and monthly-mean data.

Fig. 6 shows the meridional structures of (i) the climatological zonal mean zonal wind at 850 hPa and (ii) the southern and northern annular modes. The annular mode time series are defined as the standardized leading PCs of zonal mean 850 hPa geopotential height, Z_{850}, between 20-90 degrees latitude. Since the time series are standardized, the regression patterns shown in Fig. 6 reveal the characteristic amplitude of a one standard deviation anomaly in the annular modes. While the long-term mean circulation differs considerably between the two hemispheres, the annular modes are remarkably similar, although the NAM is slightly weaker than the SAM, consistent with the weaker climatological jet. Gerber and Vallis (2005) suggest that the meridional structure of the annular modes tend to be fairly generic, constrained largely by the geometry of the sphere and the conservation of mass and momentum.

The longitudinal correlation structures derived from the observations are not constrained to be uniform with longitude, as is the case for the dry dynamical core. Nevertheless, they are very similar from one base meridian to the next, particularly in the Southern Hemisphere. For example, Fig. 7a shows four single point covariance maps based on Z_{850} at 50°S: the covariance between Z_{850} at base points 0°, 90°E, 180°, and 90°W with all other longitudes. We have shifted the four regression plots so that the base points overlie each other at the center of the plot. Aside from slight variations in amplitude, there is remarkable uniformity of the east-west correlation structure in the midlatitudes Southern Hemisphere circulation: nearly all of the curves collapse upon each other. The correlation structures are positively correlated over a range of approximately ± 60 degrees longitude and exhibit alternating negative and positive lobes beyond that point. There is little evidence of global correlation, as is the case with the default Held and Suarez (1994) model.

Fig. 7b extends the analysis in the top panel to include averages over all base meridians for geopotential data at all latitudes. The figure is constructed as follows: (i) at a given latitude, we calculate the zonal covariance structure for all possible base meridians, as opposed to just four in Fig. 7a, (ii) we then average the resulting covariance structures after shifting them to a common base meridian, (iii) we normalize the resulting "average covariance structure" by the variance to convert to correlation coefficients, and lastly (iv) we repeat the analysis for all latitudes. The resulting "average correlation structures" for 850 hPa geopotential height are indicated by the shading in Fig. 7b. The black curve denotes the zero contour; the gray curves denote a distance of $\pm$2500 km from the base longitude to provide a sense of the sphericity of the Earth. Normalizing the covariance functions by the variance allows us to compare the longitudinal structures in the tropics and the midlatitudes on the same figure; otherwise the increase in the variance of Z_{850} with latitude (illustrated in Fig. 7c) yields much larger amplitudes in the extratropics.

At middle latitudes, positive correlations extend over a distance of approximately 2500 km outward from the base longitude. Towards the polar regions, the autocorrelations extend over much of the latitude circle due to the increasingly smaller size of the zonal ring. The austral polar regions are exceptional, in that the correlations extend not only around the circumference of the latitude circle, but also well beyond 2500 km as far equatorward as 60°S. Interestingly, tropical geopotential height is also correlated over long distances. The significant positive correlations at tropical latitudes are robust at most individual longitudes outside of the primary centers of action of ENSO (not shown). The in-phase behavior in tropical geopotential height is consistent with the dynamic constraint of weak pressure gradients at tropical latitudes (Charney 1963; Sobel et al. 2001) and will be investigated further in future work. Note that the amplitude of variations in geopotential height are more than an order of magnitude weaker in the tropics than midlatitudes, as illustrated in Fig. 7c.

The results shown in Fig. 7 are based on 10 day low pass filtered data. As discussed in

Wettstein and Wallace (2010), large-scale structures in the atmospheric circulation are increasingly prevalent at lower frequency timescales. Analogous calculations based on monthly mean data (not shown) reveal a slight extension of the region of positive correlations at all latitudes, but overall the results are qualitatively unchanged. Notably, the midlatitude correlation structure is still dominated by alternating negative and positive anomalies beyond 2500 km, with little evidence of zonally coherent motions.

How does the average correlation structure shown in Fig. 7b project onto the EOFs of the circulation? Fig. 8 characterizes the (top) "predicted" and (bottom) "actual" EOFs of zonally-varying Z_{850} calculated separately for each latitude (e.g., results at 60° N indicate the variance expressed by EOFs of Z_{850} sampled along the 60° N latitude circle). The "predicted" EOFs are found assuming the statistics of Z_{850} are zonally uniform. In this case, the results of the EOF analysis correspond to a Fourier decomposition of the flow (see discussion in Section 3), and the variance captured by each wavenumber is determined by the average correlation structure (Fig. 7b) applied to (3). Wavenumber 0 (i.e., annular mode) variability emerges as the leading predicted EOF of the flow at virtually all latitudes, but explains a much larger fraction of the variance of the flow in the tropics and polar regions than it does in middle latitudes, where wavenumbers 0, 1, 2, and 3 are of nearly equal importance. The weak amplitude of wavenumber 0 variability in middle latitudes is consistent with the lack of zonally coherent motions in the average correlation structures shown in Fig. 7b.

The "actual" EOFs are computed directly from Z_{850}, and thus do not assume that the statistics of the flow are zonally uniform. Red dots indicate when the EOF is dominated by wavenumber 0 variability, orange dots by wave 1 variability, and so forth for higher wavenumbers. (Note that for the predicted EOFs, all wavenumbers other than 0 include two modes in quadrature that account for equal variance, whereas for the actual EOFs, the two modes associated with each wavenumber are not constrained to explain the same fraction of the variance.) Comparing the top and bottom panels, it is clear that the EOFs predicted from the average correlation structure, assuming zonally-uniform statistics, provide useful insight into the true EOFs of the flow. The meridional structures of the variance explained by the leading predicted and actual EOFs are very similar: in the high latitudes and tropics, the first mode is dominated by wavenumber 0 variability and explains a much larger fraction of the flow than EOF2; in the midlatitudes, the EOFs cluster together and are largely degenerate.

The key point derived from Figs. 7 and 8 is that the "average correlation function" provides a clear sense of where the EOFs of the flow derive from robust dynamical annularity. The circulation exhibits globally coherent motions in the tropics and high latitudes, particularly in the SH high latitudes (Fig. 7), and it is over these regions that the leading EOFs predicted from the average correlation function (Fig. 8a) and from actual variations in the flow (Fig. 8b) exhibit robust wavenumber 0 variability. In contrast, the circulation does not exhibit globally coherent variations at middle latitudes (Fig. 7b), and thus both the predicted and actual EOFs of the flow are degenerate there (Fig. 8). *Annular variations in lower tropospheric geopotential height are consistent with dynamical annularity of the flow in the polar and tropical regions, but statistical annularity at middle latitudes.*

Fig. 9 explores the average correlation structure in three additional fields. Fig. 9a,b show results based on the zonal wind at 850 hPa (U_{850}), which samples the barotropic component of the circulation, and thus emphasizes the eddy-driven jet in middle latitudes. Fig. 9c,d are based on the zonal wind at 50 hPa and (U_{50}), which samples both the QBO and variations in the stratospheric polar vortices, and Fig. 9e,f, the eddy kinetic energy at 300 hPa (EKE_{300}), which samples the baroclinic annular mode (Thompson and Barnes 2014).

The most pronounced zonal correlations in U_{850} are found in two locations: (i) along 60 degrees South, where positive correlations wrap around the latitude circle, and (ii) in the deep tropics, where positive correlations extend well beyond the 2500 km isopleths. At $\sim$60 degrees South, the zonally coherent variations in the zonal flow follow from geostrophic balance and the coherence of the geopotential height field over Antarctica, as observed in Fig. 7b. In the subtropics, the far reaching correlations follow from geostrophic balance and the coherence of the geopotential height field in the tropics. At the equator, where geostrophic balance does not hold, Z_{850} exhibits globally coherent motions (consistent with weak temperature gradients in the tropics), while U_{850} becomes significantly anticorrelated at a distance. As a result, a zonally uniform annular mode dominates the EOF spectrum of Z_{850} in the tropics (Fig. 8b) whereas wavenumber 1 tends to dominate latitudinal EOF analysis of U_{850} (not shown). Neither Z_{850} (Fig. 7b) or U_{850} (Fig. 9a) exhibit zonally coherent motions at midlatitudes, where the autocorrelation function decays to zero $\sim$2500 kilometers and oscillates in the far field.

The results shown in Figs. 7b and 9a are representative of the correlation structure of geopotential height and zonal wind throughout the depth of the troposphere (e.g., very similar results are derived at 300 hPa; not shown). However, the correlation structure of the zonal flow changes notably above the tropopause, as indicated in Fig. 9c and d. Consistent with the increase in the deformation radius in the stratosphere, the scale of motions increases (note that the grey lines now indicate the $\pm$5,000 km isopleths). The most notable differences between the troposphere and stratosphere are found in the tropics, where the Quasi Biennial Oscillation (QBO) leads to an overwhelming annular signal. Marked annularity is also found in the high latitudes, in the vicinity of both extratropical polar vortices. As observed in the analysis of the tropospheric zonal wind and geopotential height, however, there is no evidence of dynamical annularity in the midlatitudes.

The average correlation structure of EKE_{300} (Fig. 9e) is notably different. Unlike Z or U, the zonal correlation of EKE is remarkably similar across all latitudes, with a slight peak in the physical scale of the correlation in the Southern Hemisphere midlatitudes where the baroclinic annular mode has largest amplitude (e.g., Thompson and Woodworth 2014). Interestingly, EKE_{300} remains positively correlated around the globe at all latitudes, albeit very weakly in the far field. The non-negative decorrelation structure leads to the dominance of a zonally uniform "annular mode" in EKE at each individual latitude poleward of 25°S, as shown in Fig. 10. However, the separation between the first and second modes (which characterize wavenumber 1 motions) is modest at most latitudes. The largest separations between the first and second EOFs EKE_{300} are found near 45°, where the top annular EOF represents about 16% of the variance, compared to about 11% for the second and third EOFs.

c. Quantifying the role of dynamical annularity in EKE_{300} with the stochastic model

At first glance, the weak separation between the first and second EOFs of EKE_{300} suggests that much of the annular signal owes itself to local correlations, i.e., statistical annularity. However, a comparison of the EOFs of the observations with those derived from the "Gaussian + baseline" model explored in Sections 2 and 3 allows us to be more quantitative about the relative role of dynamical vs. statistical annularity in the context of the baroclinic annular mode.

Fig. 11 compares (a) the zonal correlation structure and (b) EOF spectrum of the 300 hPa eddy kinetic energy at 46°S with three fits of the simple stochastic model, each designed to capture key features of the observed behavior. Recall that the model has two parameters: the width of local correlation, α, and the baseline correlation strength, β. As our goal is to focus on the relative role of dynamical annularity, characterized by the difference between the variance expressed by the top EOF (annular mode) and higher order modes, we remove one degree of freedom by requiring that the top EOF express the same fraction of variance in both the simple model and the reanalysis. Hence the first mode explains 16% of the variance for all cases in Fig. 11b. From equation (3), this condition is equivalent to keeping the total integral of the correlation structure fixed.

In the first fit (red curve, Fig. 11a), we optimize the stochastic model at short range, approximating the fall in local correlation in EKE as a Gaussian with width $\alpha = 17$ degrees. To maintain the variance expressed by the top EOF, parameter β must then be set to 0.08. This choice effectively lumps the midrange shoulder of the EKE_{300} correlation (30-100°) with the long range (100-180°), where the observed correlation drops to about 0.03. As a result, the stochastic model exhibits a stronger separation between the first and second EOFs than for EKE_{300} (red triangles vs. black squares in Fig. 11b).

An advantage of fitting the data to the simple stochastic model is that it allows us to explicitly quantify the role of dynamical annularity. Since the variance expressed by the annular mode is just the integral of correlation function (equation 3), the contribution of the long range correlation (dynamical annularity) to the total power of the annular mode is:

$$\frac{\int_0^{180} \beta \, d\lambda}{\int_0^{180} [(1-\beta)e^{-(\lambda/\alpha)^2} + \beta] \, d\lambda} \approx \frac{\beta}{\frac{\alpha(1-\beta)\sqrt{\pi}}{360} + \beta} \tag{5}$$

where we have expressed longitude λ and parameter α in degrees. For the approximation on the left hand side, we assume that $\alpha << 180$, such that the local correlation does not significantly wrap around the latitude circle. For the "red" model in Fig. 11, dynamical annularity accounts for half of the total strength of the annular mode. Given the fact that it exhibits a stronger separation between the first and second EOFs, however, this is an upper bound on the role of dynamical annularity in EKE_{300} at 46°S.

We obtain a lower bound on the dynamical annularity with the blue fit in Fig. 11a, where the correlation structure is explicitly matched at long range. To conserve the total integral, parameter α in this case must be set to 27°, effectively lumping in the shoulder between 30 and 100° with the local correlation. These parameters would suggest that dynamical annularity contributes only $1/5^{\text{th}}$ of annular mode variance. This is clearly a lower limit, however, as the separation between the first and second EOFs (Fig. 11b) is too small relative to that of EKE_{300}.

Lastly, we use both degrees of freedom of the stochastic model to find an optimal fit of the EOF spectrum, matching the variance expressed by the top two EOFs (effectively the top three, as higher order modes come in pairs). The fit, with parameters $\alpha = 23°$ and $\beta = 0.05$, is not shown in Fig. 11a (to avoid clutter), but the resulting EOF spectrum is illustrated by the green triangles in Fig. 11b. With this configuration, dynamical annularity contributes approximately $1/3^{\text{rd}}$ of the annular mode, leaving the remaining two thirds to statistical annularity associated with the local redness of the EKE. The EOF spectra of this model diverges from EKE_{300} for higher order modes, such that we should take this as a rough estimate of the true role of dynamical annularity in the Baroclinic Annular Mode.

The location of the three models (lower, optimal, and upper bounds), are marked by the black x's in Fig. 3b, to put them in context of earlier results. The fits roughly fill in the space between models X_1 and X_2, but on a lower contour where the annular mode expresses 16% of the total variance, as opposed to 20%. The rapid increase in the role of dynamical annularity (from 1/5 to 1/2) matches the rapid ascent in the importance of EOF 1 relative to EOF 2, emphasizing the utility of this ratio as an indicator of dynamical annularity.

5. Concluding Remarks

We have explored the conditions that give rise to annular patterns in Empirical Orthogonal Function analysis across a hierarchy of systems: highly simplified stochastic models, idealized atmospheric GCMs, and reanalyses of the atmosphere. Annular EOFs can arise from two conditions, which we term *dynamical annularity* and *statistical annularity*. The former arises from zonally coherent dynamical motions across all longitudes, while the latter arises from zonally coherent statistics of the flow (e.g., the variance), even in the absence of significant far field correlations. Atmospheric reanalyses indicate that both play important roles in the climate system and may aid in the interpretation of climate variability, but only dynamical annularity reflects zonally coherent motions in the circulation.

In general, dynamical annularity arises when the dynamical scales of motion approach the scale of the latitude circle. The average zonal correlation structure (e.g., Fig. 7) thus provides a robust measure of dynamical annularity. In addition, the simple stochastic model suggests that the degree of dynamical annularity in a leading EOF is indicated by the ratio of the variances explained by the first two zonal EOFs of the flow. As a rule of thumb, if the leading annular EOF explains more than twice the variance of the second EOF, then dynamical annularity plays a substantial role in the annular mode. Note, however, that this intuition does not necessarily apply to two-dimensional EOFs in latitude-longitude space, where coherence of meridional variability can lead to dominance of an annular EOF, even when there is explicitly no dynamical annularity (e.g., Gerber and Vallis 2005).

Annular EOFs always – at least partially – reflect statistical annularity of the circulation; zonally coherent motions necessarily imply some degree of zonal coherence. Far field correlation in the average zonal correlation structure robustly indicates dynamical annularity, but quantification of the statistical annularity requires further analysis, either comparison of the zonal correlation at different base points (e.g., Fig. 7a) or comparison of the predicted and observed zonal EOFs (e.g., Figs. 8 and 10). The localization of the North Pacific and North Atlantic storm tracks limits the utility of the zonal correlation structure in the Northern Hemisphere troposphere. But the Southern Hemisphere tropospheric circulation is remarkably statistically annular, such that one can predict the full EOF spectrum from the average correlation structure alone.

As discussed in Deser (2000) and Ambaum et al. (2001), the observed geopotential height field does not exhibit robust far field correlations beyond $\sim$60 degrees longitude in the midlatitudes. However, the geometry of the sphere naturally favors a high degree of zonal coherence at higher latitudes. Hence, the northern and southern annular modes do not arise from dynamical annularity in the middle latitude tropospheric circulation, but derive a measure of dynamical annularity from the coherence of their polar centers of action in the geopotential height field. The dynamical annularity of the annular mode extends to the zonal wind field at high latitudes in the Southern Hemisphere, but less so in the Northern Hemisphere. Other regions where dynamical annularity plays a seemingly important role in the circulation include:

i. the tropical geopotential height field, presumably because temperature gradients must be weak in this region (e.g., Charney 1963),

ii. the tropospheric zonal flow near $\sim$15 degrees latitude; these features arises via geostrophy and the dynamic annularity of the tropical Z field.

iii. the zonal wind field in the equatorial stratosphere, which reflects the QBO,

iv. the eddy kinetic energy in the midlatitude Southern Hemisphere, consistent with the baroclinic annular mode and the downstream development of wave packets in the austral stormtrack (Thompson et al. submitted). The dynamical annularity of the eddy activity is surprising given the lack of dynamic annularity in the midlatitude barotropic jets, which are intimately connected with the eddies through the baroclinic lifecycle.

The annular leading EOFs of the midlatitude flow have been examined extensively in previous work, but to our knowledge, the annular nature of tropical tropospheric Z has received less attention. We intend to investigate this feature in more detail in a future study.

Acknowledgments.

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APPENDIX

Technical details of the stochastic models

The stochastic models in Section 2 are, in a sense, constructed in reverse, starting with the desired result. We begin with the correlation structure f, as shown in Fig. 1c, and project it onto cosine modes as in (3). This gives us the EOF spectra shown in Fig. 2c, i.e., how much variance (which we now denote v_k) should be associated with each mode of wavenumber k. Note that not all correlation structures are possible. A sufficient criteria, however, is that the projection of every cosine mode onto f is non-negative (i.e., all $v_k \geq 0$).

Realizations of the models, as shown in 1a and b, are constructed by moving back into grid space,

$$X(\lambda, j) = v_k^{1/2}\delta_{0,j} + \sum_{k=1}^{\infty}(2v_k)^{1/2}[\delta_{k1,j}\sin(k\lambda) + \delta_{k2,j}\cos(k\lambda)]. \tag{A1}$$

where all the $\delta_{k,j}$ are independent samples from a Normal distribution with unit variance and λ is given in radians. In practise only the top 15 wavenumbers are needed, as the contribution of higher order modes becomes trivial.

Note that it is possible to construct an infinite number of stochastic systems which have the same correlation structure f. We have take a simple approach by using the Normal distribution to introduce randomness. Any distribution with mean zero could be used, which would impact the variations in individual samples – and so the convergence of the system in j – but not the statistical properties in the limit of infinite sampling.

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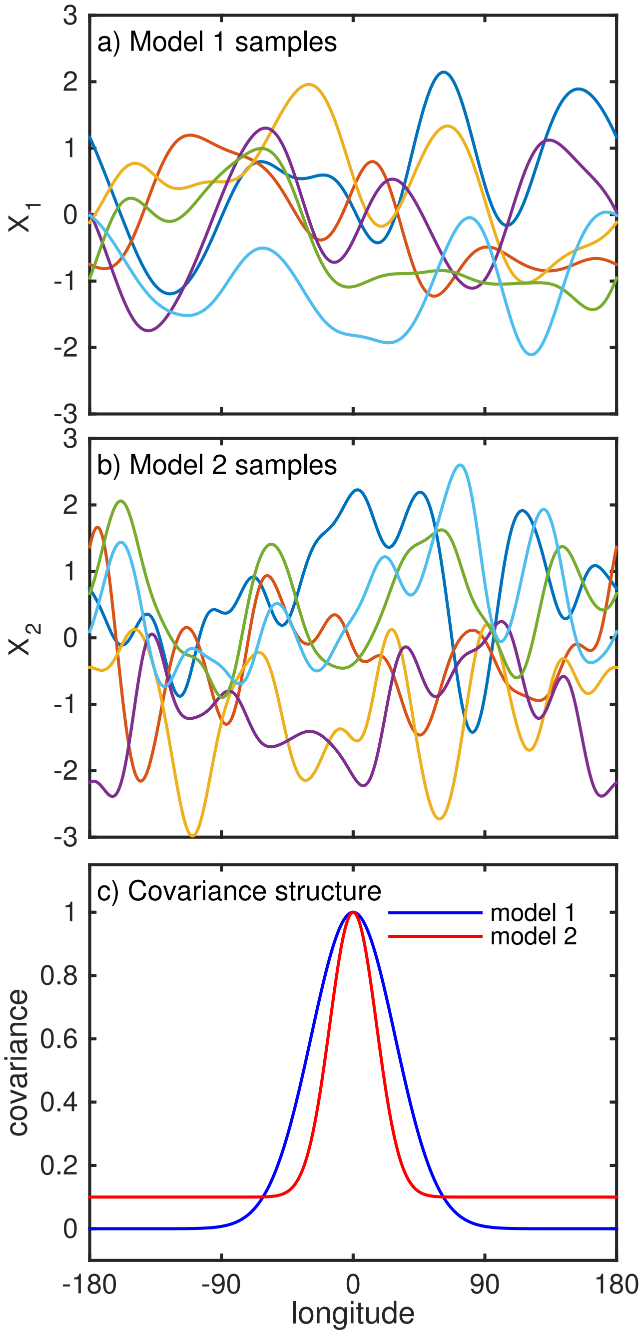

FIG. 1. Two stochastic models of variability in longitude. (a) and (b) illustrate sample profiles from models X_1 and X_2, respectively. The y-axes are unitless, as each model has been designed to have unit variance. (c) shows $\mathrm{cov}_X(0, \lambda)$ for each model, the covariance between variability at each longitude with that at $\lambda = 0$. As the statistics are annular, the covariance structure can be fully characterized by this one sample, i.e., $\mathrm{cov}_X(\lambda_1, \lambda_2) = \mathrm{cov}_X(0, |\lambda_1 - \lambda_2|)$.

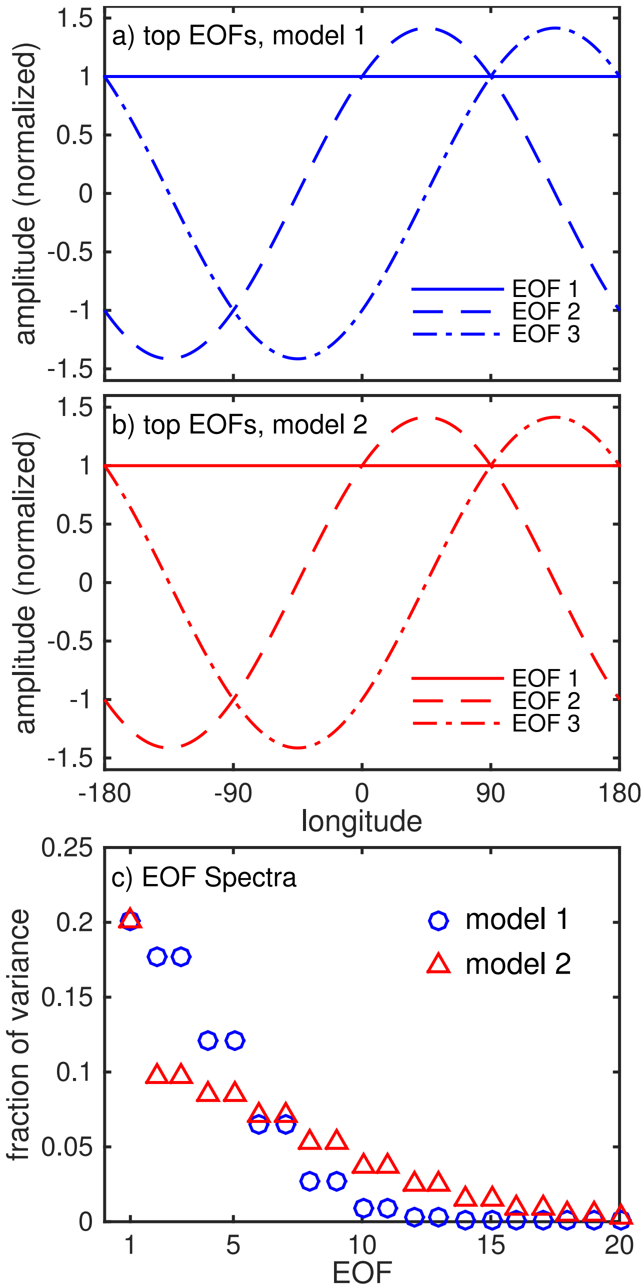

FIG. 2. The EOF structure of the two stochastic models. (a) and (b) show the top three EOFs for models 1 and 2, respectively, normalized to have unit variance. In the limit of infinite sampling, the EOF patterns from the two models are identical. (c) The models' EOF spectra, marking the fraction of the total variance associated with each of the top 20 EOFs.

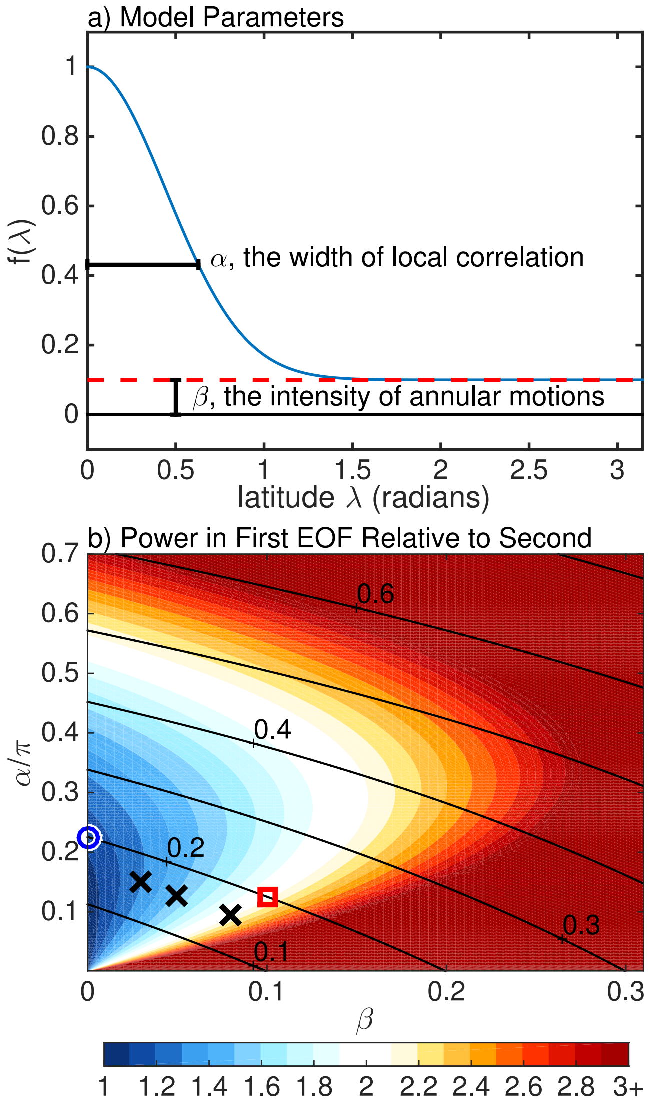

FIG. 3. The impact of local vs. annular correlation in the "Gaussian + baseline" family of stochastic models. (a) illustrates the parameters α and β which characterize the correlation function $f(\lambda)$ for each model. (b) maps out the variance expressed by the first EOF (black contours) and the ratio of the variance expressed by the first EOF to that of the second (color shading) as a function of α and β. The first EOF is always annular, and the second always a wavenumber 1 pattern. The blue and red markers show the location of models X_1 and X_2 (illustrated in Figs. 1 and 2) in parameter space, respectively; both fall along the same black contour, as their top EOF expresses 0.2 of the total variance. The black x's will be discussed in the context of Fig. 11

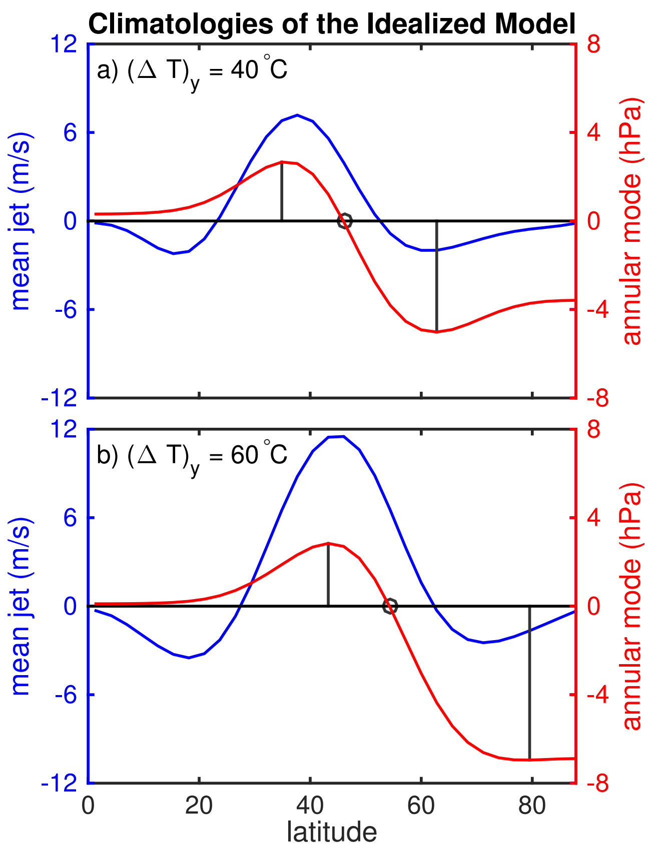

FIG. 4. The mean jet structure and annular modes of the Held and Suarez (1994) model for the (a) $(\Delta T)_y = 40$ and (b) $(\Delta T)_y = 60°\text{C}$ integrations. The jet is characterized by the time mean 850 hPa winds (blue lines, corresponding with the left y-axes), and the annular mode is the first EOF of daily, zonal mean SLP (red, right y-axes), normalized to indicate the strength of 1 standard deviation anomalies. The latitudes of the node, equatorward and poleward lobes of the annular mode are highlighed, and correspond with the analysis in Fig. 5.

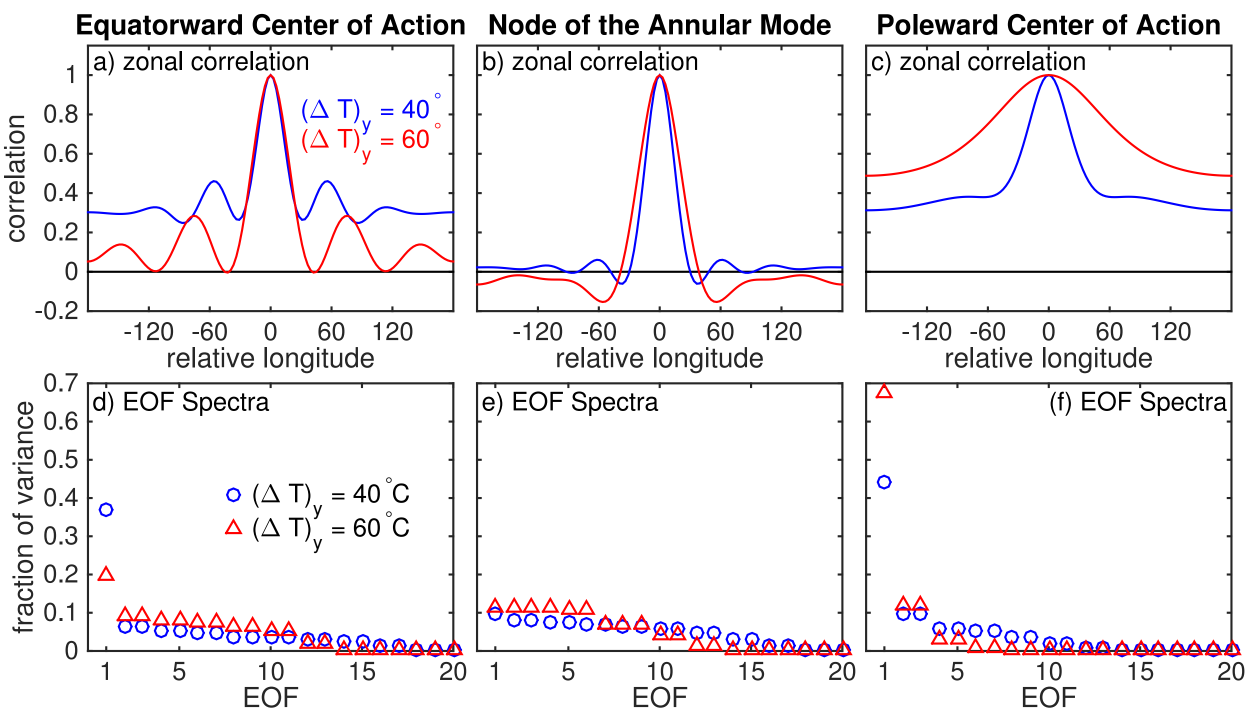

FIG. 5. Characterizing the zonal structure of 10 day pass filtered SLP anomalies in the Held and Suarez (1994) model. (a,d) and (c,f) show analysis based at the latitude of the equatorward and poleward centers of action of the annular mode, respectively, while (b,e) show analysis based at the nodes of the annular mode. (a,b,c) show the zonal correlation structure $f(\lambda)$ and (d,e,f) the fraction of variance associated with each of the top 20 EOFs for the integrations with (blue) $(\Delta T)_y = 40$ and (red) $(\Delta T)_y = 60^\circ$ C.

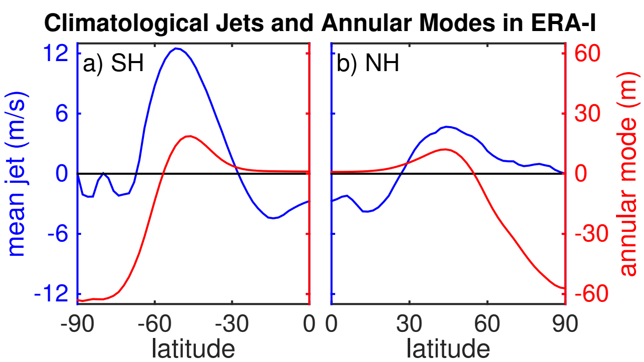

FIG. 6. The same as Fig. 4, but for the (a) Southern and (b) Northern Hemispheres in ECWMF Interim reanalysis, based on the period 1979-2013. To avoid interpolation over mountainous regions, the annular modes are defined in terms of daily, zonal mean 850 hPa geopotential height, Z_{850}, instead of SLP.

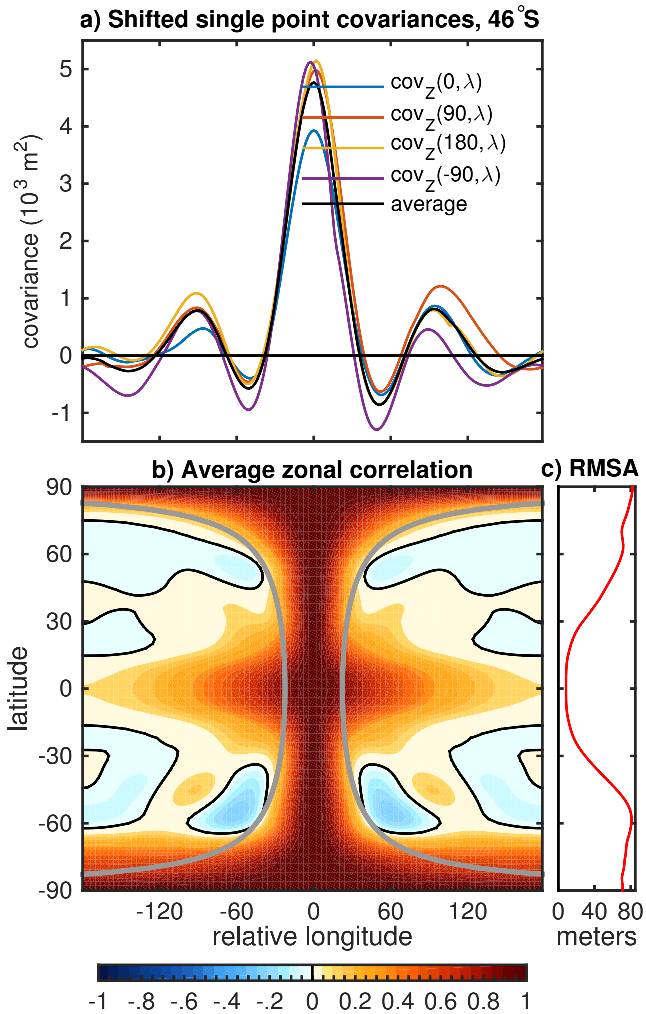

FIG. 7. Characterizing the longitudinal correlation structure of 10 day low pass filtered 850 hPa geopotential height in ERA-Interim. (a) Sample single point correlation maps at 46°S (the equatorward center of action of the SAM), shifted so that base points line up. The black line is the mean of the four curves, an "average single point correlation map". (b) The average zonal correlation structure of 10 day low pass filtered Z_{850} as a function of latitude. The contour interval is 0.05, with black contours marking zero correlation, and gray lines indicate a separation of 5000 km, to provide a sense of geometry on the sphere. (c) The root mean square amplitude of 10 day low pass filtered Z_{850} anomalies.

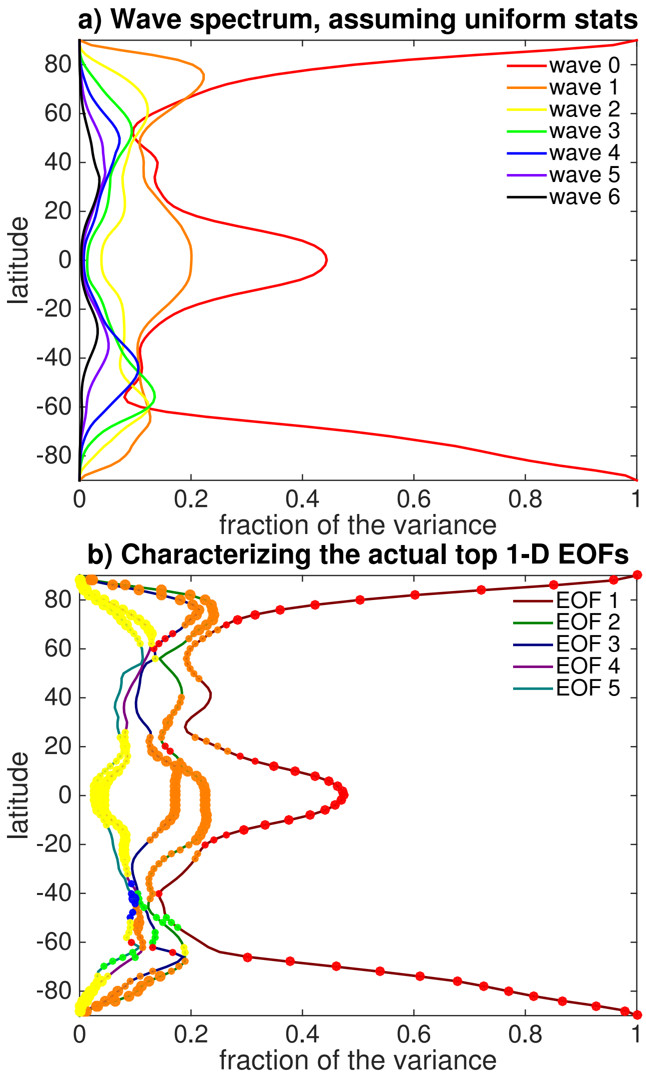

FIG. 8. A comparison of predictions based on zonally uniform statistics to the actual zonal EOF structure of 10 day low pass filtered Z_{850}. (a) For each latitude, the fraction of variance associated with wavenumbers 0 to 6, given the average zonal correlation structure in Fig. 7b and assuming zonally uniform statistics (see text for details). (b) Again for each latitude, the fraction of variance associated with the top five 1-D longitudinal EOFs, but now based on the full flow. Large (small) colored dots indicate when a given wavenumber dominates more than 75% (50%) of the power in the EOF, the color identifying the respective wavenumber with the color convention in (a), i.e., red=wave 0, orange=wave 1.

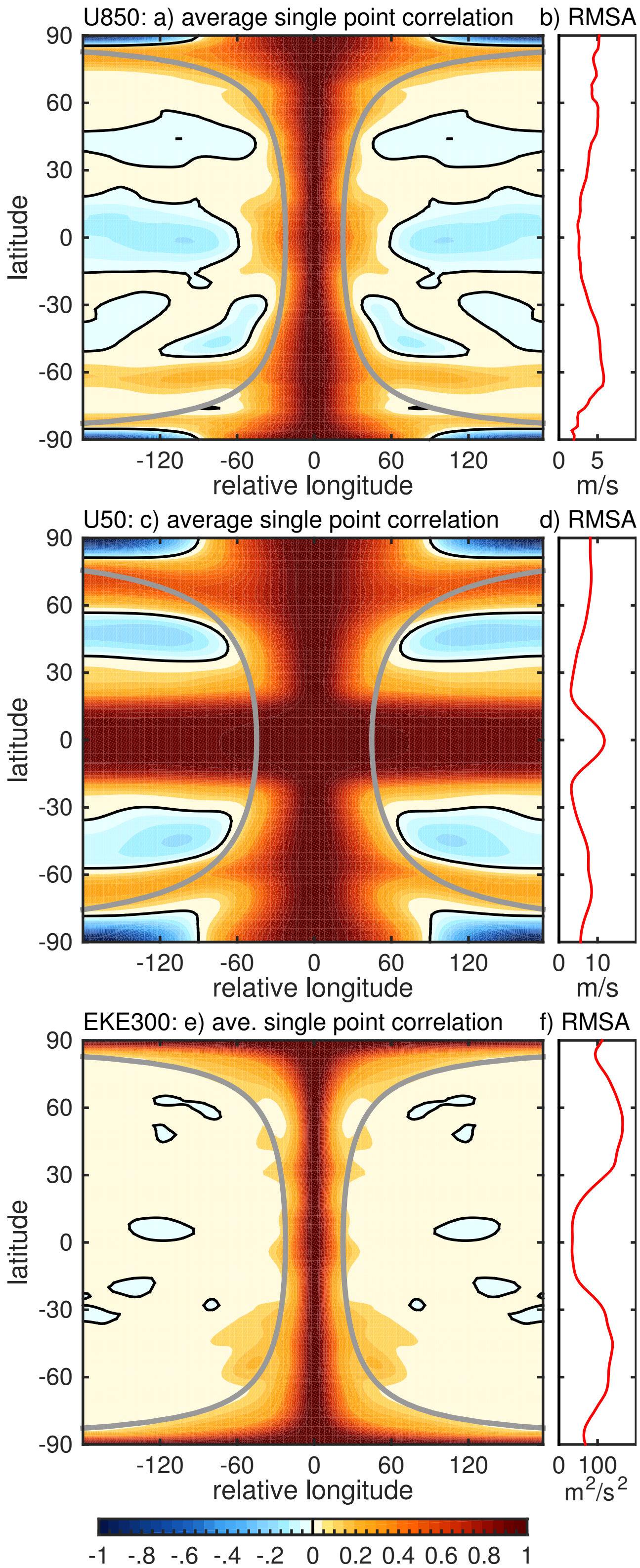

FIG. 9. The average correlation structure of (a) zonal wind at 850 hPa, (c) zonal wind at 50 hPa, and (e) eddy kinetic energy at 300 hPa. As in Fig. 7b, thin black contours mark zero correlation and the thick gray contours give a sense of sphericity, marking a separation of 5000 km as a function of latitude in (a) and (e) and a distance of 10000 km in (c). Panels (b), (d), and (f) show the root mean square amplitude of variations as a function latitude for each variable, respectively.

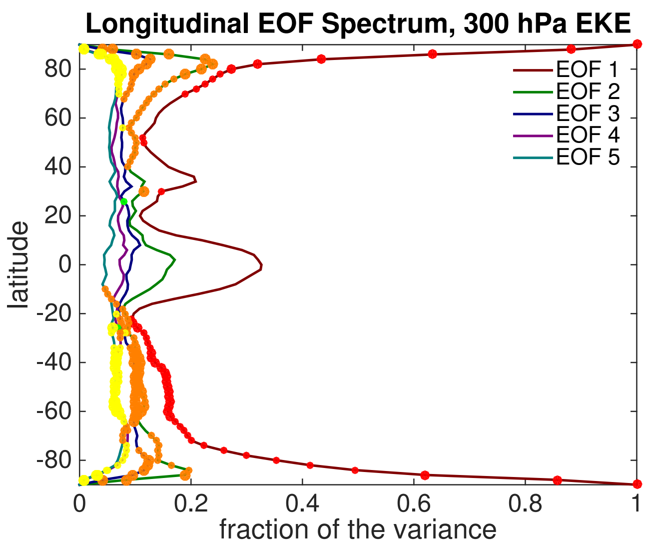

Fig. 10. The same as in Fig. 8b, but for eddy kinetic energy at 300 hPa. Zonal asymmetry in the statistics lead to substantial mixing between wavenumbers in the Northern Hemisphere (outside the polar cap) and tropics, such no single wavenumber dominates each EOF. Statistical annularity in the Southern Hemisphere, however, leads to a clearly order spectrum poleward of 25°S, dominated by an annular (wavenumber 1) mode at all latitudes.

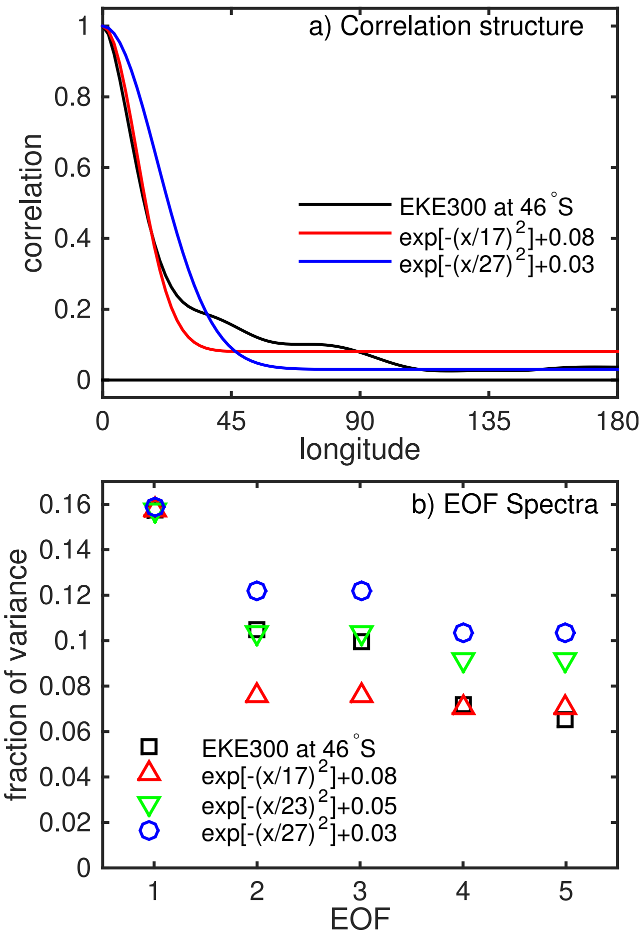

FIG. 11. (a) Comparison between the average longitudinal correlation structure of EKE_{300} at 46°S and two possible fits with the Gaussian + baseline model of Section 3. As detailed in the text, the first fit (red) is optimized to capture the initial decay in correlation, while the second fit (blue) is optimized for the long range correlation baseline. (b) The 1-dimensional EOF spectra of EKE_{300} at 46°S, compared against the spectrum for the two fits of the Gaussian + baseline model shown in (a), and a third model with parameters $\alpha = 23°$ and $\beta = 0.05$, as discussed in the text.